

# FINITE SIZE OF HADRONS AND BOSE-EINSTEIN CORRELATIONS

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## WARNING

### CORRELATION BETWEEN MOMENTA OF TWO IDENTICAL HADRONS

$$C(p_1, p_2) \equiv \frac{N(p_1, p_2)}{N(p_1)N(p_2)} - 1 \quad (1)$$

### IS USUALLY ANALYZED USING THE STANDARD FORMULA

$$C(p_1, p_2) = \frac{\tilde{w}(P_{12}; Q)\tilde{w}(P_{12}; -Q)}{w(p_1)w(p_2)} = \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1)w(p_2)} \quad (2)$$

WHERE  $w(p.x)$  IS THE SINGLE-PARTICLE  
MOMENTUM-SPACE DISTRIBUTION;

$$\tilde{w}(P_{12}; Q) = \int dx e^{iQx} w(P_{12}; x); \quad w(p) = \int dx w(p; x)$$
$$P_{12} = (p_1 + p_2)/2; \quad Q = p_1 - p_2,$$

THIS PROCEDURE IS - IN GENERAL - INCORRECT.

## TWO PARTICLE CORRELATIONS

LET  $W(p_1, p_2; x_1, x_2)$  BE THE MOMENTUM AND SPACE "DISTRIBUTION" OF TWO PARTICLES ("SOURCE FUNCTION"). IF PARTICLES ARE IDENTICAL, THE OBSERVED MOMENTUM DISTRIBUTION IS

$$\begin{aligned}\Omega(p_1, p_2) &= \int dx_1 dx_2 W(p_1, p_2; x_1, x_2) + \\ &+ \int dx_1 dx_2 e^{i(x_1 - x_2)Q} W(P_{12}, P_{12}; x_1, x_2) \equiv \\ &\equiv \Omega_0(p_1, p_2) [1 + C(p_1, p_2)]\end{aligned}\quad (3)$$

WHERE  $P_{12} = (p_1 + p_2)/2$ ,  $Q = p_1 - p_2$ , AND

$$\Omega_0(p_1, p_2) = \int dx_1 dx_2 W(p_1, p_2; x_1, x_2) \quad (4)$$

ONE SEES THAT  $C(p_1, p_2)$  CONTAINS INFORMATION ONLY ON THE DISTRIBUTION OF  $x_1 - x_2$ .

# NO MOMENTUM-SPACE CORRELATIONS

**IF THERE ARE NO MOMENTUM AND SPACE CORRELATIONS,**

$$W(p_1, p_2; x_1, x_2) = w(p_1, x_1)w(p_2, x_2)$$

**THEN**  $\Omega(p_1, p_2) = w(p_1)w(p_2) + |\tilde{w}(P_{12}, Q)|^2$ ,

**WHERE**  $\tilde{w}(P_{12}, Q) = \int dx w(P_{12}, x)e^{ixQ}$ .

**THUS THE CORRELATION FUNCTION IS**

$$C_2(p_1, p_2) = \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1)w(p_2)} \geq 0!!!! \quad (5)$$

**THIS IS THE COMMONLY USED FORMULA.**

**FROM  $\tilde{w}(P_{12}, Q)$  ONE CAN RECOVER  $w(P_{12}, x)$ .**

**BUT:** *THIS IS VALID ONLY IF THERE ARE NO MOMENTUM AND SPACE CORRELATIONS.*

# CORRELATIONS IN SPACE

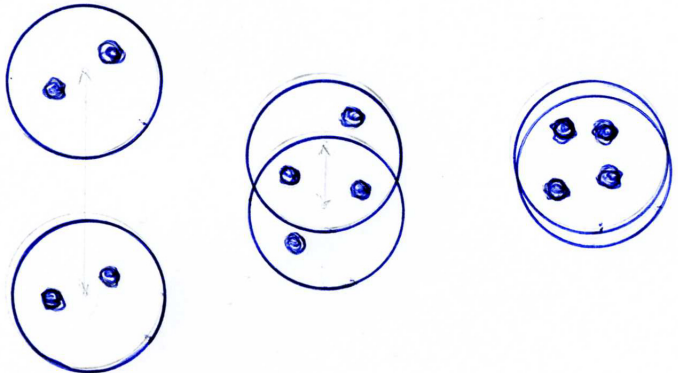
**IDEA: WHEN PIONS ARE TOO CLOSE TO EACH OTHER THEY ARE *NOT* PIONS ANYMORE!!! ( BECAUSE THEIR CONSTITUENTS ARE MIXING AND THEIR WAVE FUNCTIONS ARE NOT WELL-DETERMINED).**

**SINCE HBT EXPERIMENTS MEASURE QUANTUM INTERFERENCE BETWEEN THE WAVE FUNCTIONS OF PIONS THEY CANNOT SEE PIONS TOO CLOSE TO EACH OTHER.**

**THEREFORE  $W(P_{12}, P_{12}; x_1, x_2)$  MUST VANISH AT SMALL  $|x_1 - x_2|$ , IMPLYING CORRELATION BETWEEN POSITIONS OF TWO PIONS.**

PICTURE

# MIXING OF QUARKS



## CORRELATIONS IN SPACE

**Repeat:**  $W(P_{12}, P_{12}; x_1, x_2)$  **MUST VANISH AT**  $|x_1 - x_2| = 0$ ,  
**IMPLYING CORRELATION BETWEEN POSITIONS OF**  
**TWO PIONS. THIS IS THE NECESSARY CONSEQUENCE**  
**OF THE FUNDAMENTAL PROPERTY OF HADRONS:**  
**THEY ARE NOT POINT-LIKE.**

**THUS THE TWO-PION DISTRIBUTION IS OF THE**  
**FORM**

$$W(P_{12}, P_{12}; x_1, x_2) = w(P_{12}; x_1)w(P_{12}; x_2)[1 - \Delta(x_1 - x_2)]. \quad (6)$$

**THE CORRELATION FUNCTION:**

$$C(p_1, p_2) = \frac{|\tilde{w}(P_{12}, Q)|^2}{w(p_1)w(p_2)} - C_{corr}(p_1, p_2);$$
$$C_{corr} = \frac{\int dx_1 dx_2 e^{i(x_1 - x_2)Q} w(P_{12}; x_1)w(P_{12}; x_2)\Delta(x_1 - x_2)}{w(p_1)w(p_2)} \quad (7)$$

# EXAMPLE

FOR ILLUSTRATION, TAKE

$$\Delta(x_1 - x_2) = \Theta[r^2 - |\vec{x}_1 - \vec{x}_2|^2 - (t_1 - t_2)^2];$$

$$w(P, x) = e^{-|\vec{x}|^2/R^2} e^{-t^2/\tau^2} f(P)$$

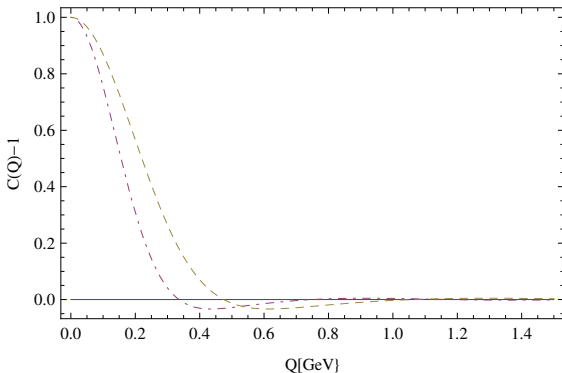


Figure: Oscillating two-pion correlation function.  $R = r = \tau = 1$  fm.



# DATA L3

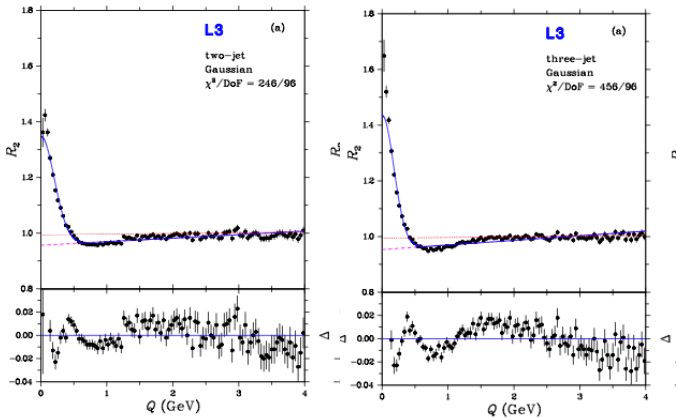


Figure: L3 data for two-jet and three-jet events.

# DATA CMS 1

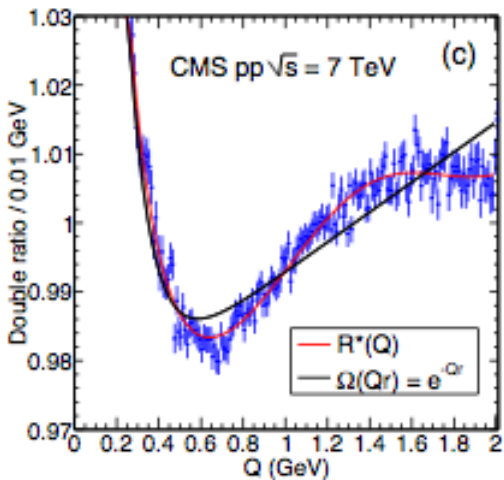


Figure: Two-pion correlation function from CMS (pp at 7 TeV)

# DATA CMS 2

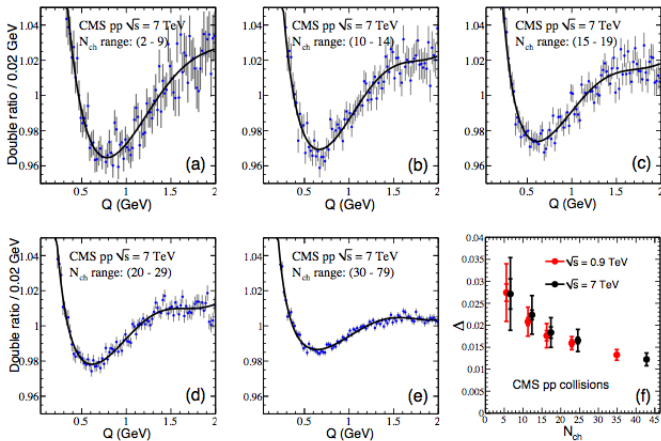


Figure: Two-pion correlation function for various multiplicities from CMS (pp at 7 TeV)

# COMMENTS

**(i) Our qualitative argument shows that the observed oscillations of the HBT correlation function are not accidental but reflect the fundamental fact that hadrons are not point-like. Therefore they deserve more attention in data analysis. It seems that the effect simply **MUST BE THERE** and the real experimental challenge is to determine its size.**

**(ii) More serious calculations, as well as a detailed comparison with data is clearly needed and are in progress (together with W.Florkowski)**

**(a) It is necessary to formulate a consistent phenomenological framework where the measured HBT radii are correctly described.**

**(b) Including other space correlations (e.g. clusters) may also be of interest.**