Ghosts in Keldysh-Schwinger Formalism

Alina Czajka

Institute of Physics,
Jan Kochanowski University, Kielce, Poland

in collaboration with St. Mrówczyński

based on: A. Czajka & St. Mrówczyński, Phys. Rev. D **89**, 085035 (2014)

Outline

1. Motivation
2. Keldysh-Schwinger formalism
3. Green's functions of gluons
4. Generating functional
5. Slavnov-Taylor identities
6. Green's functions of ghosts
7. Application - polarization tensor
8. Conclusions

Motivation

- QCD computations in covariant gauges are usually much simpler than those in physical ones like the Coulomb gauge.

- Covariant gauges require ghosts to compenasate unphysical degrees of freedom.

How to introduce ghosts in the Keldysh-Schwinger formalism?

What is the Green's function of free ghosts?

Keldysh–Schwinger formalism

Description of non-equilibrium many-body systems

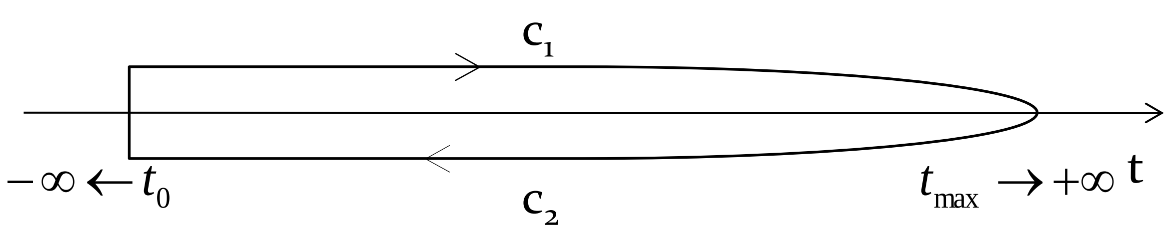
Contour Green function of gauge field

$$i\mathcal{D}_{ab}^{\mu\nu}(x,y) \overset{\text{def}}{=} \left\langle \tilde{T} A_a^{\mu}(x) A_b^{\nu}(y) \right\rangle$$

$$\langle \ldots \rangle = \mathrm{Tr}[\hat{\rho}(t_0)\ldots]$$

$\tilde{T}$ - ordering along the contour

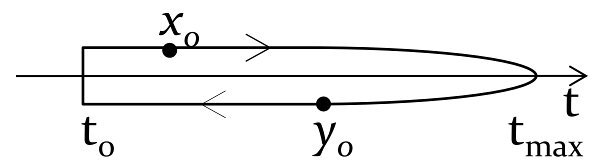
$$\tilde{T} A(x)B(y) = \Theta(x_0, y_0) A(x)B(y) \pm \Theta(y_0, x_0) B(y)A(x)$$



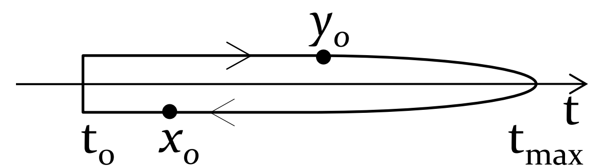
Keldysh–Schwinger formalism

Contour Green's function includes 4 Green's functions with real time arguments:

$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{>}(x,y) = \left\langle A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$

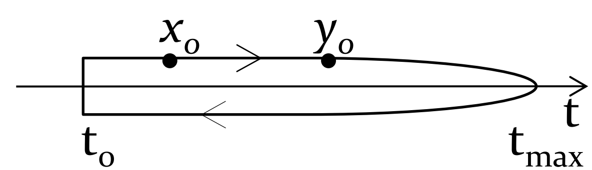


$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{<}(x,y) = \left\langle A^{\nu}_{b}(y) A^{\mu}_{a}(x) \right\rangle$$

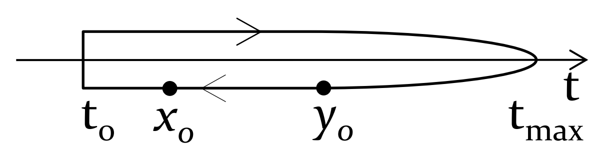


$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{c}(x,y) = \left\langle T^{c} A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$

Chronological time ordering



$$\left(\mathcal{D}^{\mu\nu}_{ab}\right)^{a}(x,y) = \left\langle T^{a} A^{\mu}_{a}(x) A^{\nu}_{b}(y) \right\rangle$$



Anti-chronological time ordering

Retarded, advanced & symmetric Green's functions

$$\boxed{\mathcal{D}^{+}(x,y)} = \Theta(x_0 - y_0)\left(\mathcal{D}^{>}(x,y) - \mathcal{D}^{<}(x,y)\right)$$

$$\boxed{\mathcal{D}^{-}(x,y)} = \Theta(y_0 - x_0)\left(\mathcal{D}^{<}(x,y) - \mathcal{D}^{>}(x,y)\right)$$

$$\boxed{\mathcal{D}^{sym}(x,y)} = \mathcal{D}^{>}(x,y) + \mathcal{D}^{<}(x,y)$$

Meaning of the functions

$\mathcal{D}^{<,>}(x,y)$

- phase-space density
- mass-shell constraint
- real particles

$\mathcal{D}^{\pm}(x,y)$

- retarded & advanced propagator
- no mass-shell constraint
- virtual particles

Meaning of the functions

The Green's function $\mathcal{D}(x,y)$ are gauge dependent

Physical results obtained from Green's functions must be gauge independent

For example

The poles of $\mathcal{D}(x,y)$ - disperssion relations – are gauge independent

Green's functions of free gluon field

Feynman gauge $\alpha = 1$

$$D^{>}(p) = \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab}\left[\delta(E_p - p_0)[n_g(\mathbf{p})+1] + \delta(E_p + p_0)n_g(-\mathbf{p})\right]$$

$$D^{<}(p) = \frac{i\pi}{E_p} g_{\mu\nu}\delta^{ab}\left[\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)[n_g(-\mathbf{p})+1]\right]$$

$$D^{c}(p) = -g_{\mu\nu}\delta^{ab}\left[\frac{1}{p^2 + i0^+} - \frac{i\pi}{E_p}\left(\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)n_g(-\mathbf{p})\right)\right]$$

$$D^{a}(p) = g_{\mu\nu}\delta^{ab}\left[\frac{1}{p^2 - i0^+} + \frac{i\pi}{E_p}\left(\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)n_g(-\mathbf{p})\right)\right]$$

$n_g(\mathbf{p})$ - gluon distribution function

Green's functions of free ghosts

$$\left.\begin{array}{l}\Delta^{>}(p)\\ \Delta^{<}(p)\\ \Delta^{c}(p)\\ \Delta^{a}(p)\end{array}\right\} = ?$$

How to get Green's function of free ghosts?

Ghost sector should be determined by the gauge symmetry of the theory!

$$A_\mu^a \to \left(A_\mu^a\right)^U = A_\mu^a + f^{abc}\omega^b A_\mu^c - \frac{1}{g}\partial_\mu \omega^a$$

↓ gauge symmetry of the theory

Slavnov-Taylor identities

Generating functional

$$W_0[J,\chi,\chi^*] = N_0 \int_{BC} \mathcal{D}A\, \mathcal{D}c\, \mathcal{D}c^*\, e^{i\int_C d^4x\, \mathcal{L}_{\text{eff}}(x)}$$

all fields are on the contour

boundary conditions:
the fields are fixed in $t = -\infty \pm i0^+$

Lagrangian:

$$\mathcal{L}_{\text{eff}}(x) = -\frac{1}{4} F_a^{\mu\nu} F_{\mu\nu}^a + \bar{\psi}(i\gamma_\mu D^\mu - m)\psi - \frac{1}{2\alpha}(\partial^\mu A_\mu^a)^2$$
$$- c_a^*(\partial^\mu \partial_\mu \delta_{ab} - g\partial^\mu f^{abc} A_\mu^c) c_b + J_\mu^a A_a^\mu + \chi_a^* c_a + \chi_a c_a^*$$

$$W[J,\chi,\chi^*] = N \int DA'\, Dc'\, Dc^{*\prime}\, DA''\, Dc''\, Dc^{*\prime\prime}$$
$$\times \rho[A',c',c^{*\prime} | A'',c'',c^{*\prime\prime}]\, W_0[J,\chi,\chi^*]$$

density matrix

Generating functional

$$W[J,\chi,\chi^*] = N\int DA' Dc' Dc^{*\prime} DA'' Dc'' Dc^{*\prime\prime} \times \rho[A',c',c^{*\prime}|A'',c'',c^{*\prime\prime}]\, W_0[J,\chi,\chi^*]$$

The full Green's function can be generated through

$$i\mathcal{D}^{ab}_{\mu\nu}(x,y) = (-i)^2 \frac{\delta^2}{\delta J^a_\mu(x)\delta J^b_\nu(y)} W[J,\chi,\chi^*]\Big|_{J=\chi=\chi^*=0}$$

density matrix $\rho[A',c',c^{*\prime}|A'',c'',c^{*\prime\prime}]$ is not specified

the explicit form of the functional and the Green's function cannot be found

The functional provides various relations among Green's functions.

General Slavnov-Taylor identity

$$W[J,\chi,\chi^*] = N\int_{BC} \mathcal{D}A\,\Delta[A]e^{i\int_C d^4x\,\mathcal{L}(x)}$$

analog of the Fadeev-Popov determinant

$$\Delta[A] \equiv \int_{BC} \mathcal{D}c\,\mathcal{D}c^*\,e^{-i\int_C d^4x\left(-c_a^*(\partial^\mu\partial_\mu\delta_{ab} - g\partial^\mu f^{abc}A_\mu^c)c_b + \chi_a^* c_a + \chi_a c_a^*\right)}$$

The invariance of $W[J,\chi,\chi^*]$ under the transformations

$$A_\mu^a \to \left(A_\mu^a\right)^U = A_\mu^a + f^{abc}\omega^b A_\mu^c - \frac{1}{g}\partial_\mu\omega^a$$

leads to

$$\left\{ i\partial^\mu_{(y)} \frac{\delta}{\delta J_d^\mu(y)} - \int_C d^4x\, J_a^\mu(x)\left(\partial_\mu^{(x)}\delta^{ab} + igf^{abc}\frac{\delta}{\delta J_c^\mu(x)}\right) M_{bd}^{-1}\left[\frac{1}{i}\frac{\delta}{\delta J}\middle| x,y\right]\right\} W[J,\chi,\chi^*] = 0$$

Slavnov-Taylor identity for gluon Green's function

$$\frac{\delta}{\delta J_e^{\nu}(z)} \left\{ i\partial^{\mu}_{(y)} \frac{\delta}{\delta J_d^{\mu}(y)} - \int_C d^4x\, J_a^{\mu}(x) \left(\partial_{\mu}^{(x)} \delta^{ab} + igf^{abc} \frac{\delta}{\delta J_c^{\mu}(x)} \right) M_{bd}^{-1} \left[\frac{1}{i} \frac{\delta}{\delta J} \middle| x, y \right] \right\} W[J, \chi, \chi^*] = 0$$

$$J = \chi = \chi^* = 0$$

$$-p^{\mu} \mathcal{D}_{\mu\nu}^{ab}(p) = p_{\nu} \Delta_{ab}(-p)$$

free ghosts Green's function

The longitudinal component of the gluon Green's function is free.

Ghost functions

$$- p^{\mu} D^{ab}_{\mu\nu}(p) = p_{\nu}\Delta_{ab}(-p)$$

$$\Delta^{>}(p) = -\frac{i\pi}{E_p}\delta^{ab}\left[\delta(E_p - p_0)[n_g(\mathbf{p})+1] + \delta(E_p + p_0)n_g(-\mathbf{p})\right]$$

$$\Delta^{<}(p) = -\frac{i\pi}{E_p}\delta^{ab}\left[\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)[n_g(-\mathbf{p})+1]\right]$$

$$\Delta^{c}(p) = \delta^{ab}\left[\frac{1}{p^2+i0^+} - \frac{i\pi}{E_p}\left(\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)n_g(-\mathbf{p})\right)\right]$$

$$\Delta^{a}(p) = -\delta^{ab}\left[\frac{1}{p^2-i0^+} + \frac{i\pi}{E_p}\left(\delta(E_p - p_0)n_g(\mathbf{p}) + \delta(E_p + p_0)n_g(-\mathbf{p})\right)\right]$$

$n_g(\mathbf{p})$ - gluon distribution function

Application - polarization tensor

The poles of $\mathcal{D}(x, y)$ give disperssion relations of quasiparticles

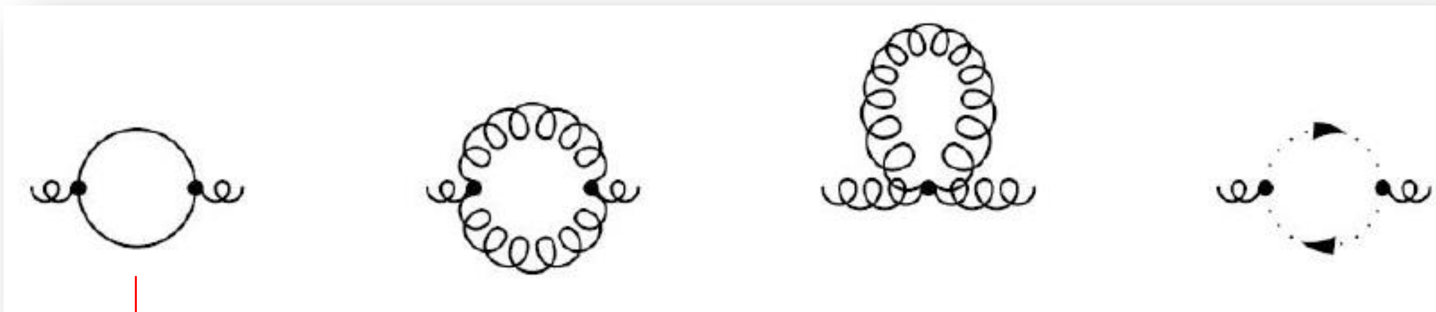
Dyson – Schwinger equation

$$\mathcal{D} = D - D\Pi\mathcal{D}$$

$$\mathcal{D}^{-1} = D^{-1} + \Pi$$

To get disperssion relations one needs the polarization tensor

Contributions to polarization tensor

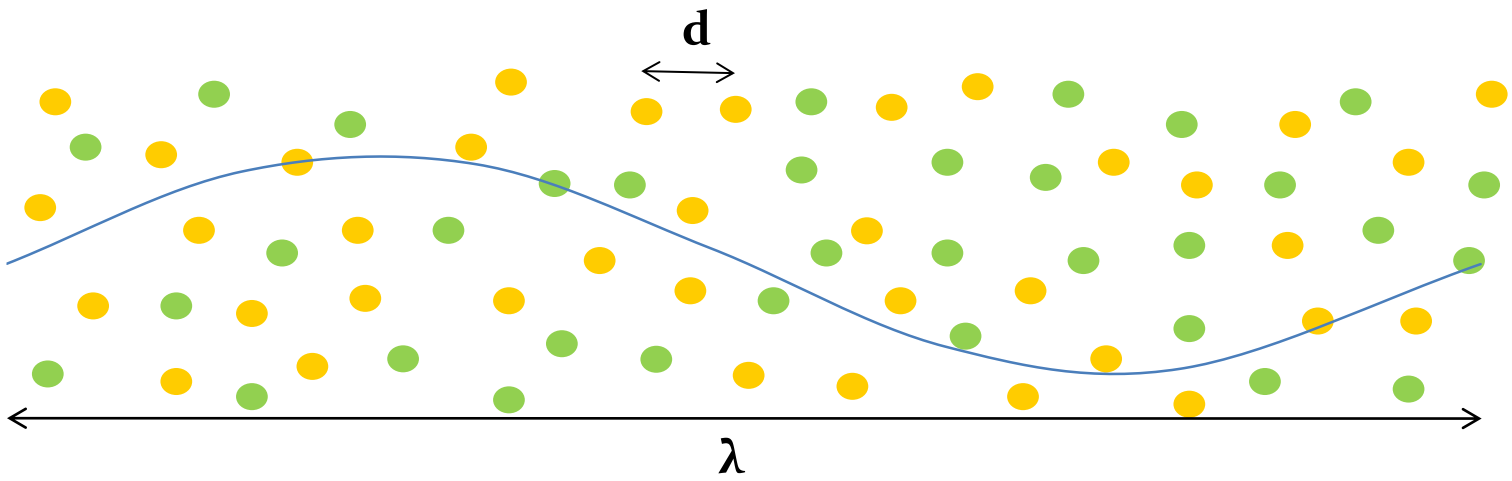


$$\Pi_{ab}^{\mu\nu}(x,y) = -ig^2 N_c \delta_{ab} \, \mathrm{Tr}[\gamma^\mu S(x,y) \gamma^\nu S(y,x)]$$

quark-loop contribution to <u>contour</u> polarization tensor

$S(x,y)$ - fermion contour Green's function

Hard Loop Approximation



Wavelength of a quasi-particle is much bigger than inter-particle distance in the plasma:

$$\lambda >> d$$

$$k^{\mu} << p^{\mu}$$

Polarization tensor

$$\Pi^{\mu\nu}_{ab}(k) = g^2\delta_{ab}\int\frac{d^3p}{(2\pi)^3}\,\frac{f(\mathbf{p})}{E_p}\,\frac{k^2p^\mu p^\nu - [p^\mu k^\nu + k^\mu p^\nu - g^{\mu\nu}(k\cdot p)](k\cdot p)}{(k\cdot p + i0^+)^2}$$

distribution function

$$f(\mathbf{p}) \equiv 2N_c n_g(\mathbf{p}) + n_q(\mathbf{p}) + \bar{n}_q(\mathbf{p})$$

(vacuum effect is subtracted)

- **symmetric** $\Pi^{\mu\nu}(k) = \Pi^{\nu\mu}(k)$
- **transversal** $k_\mu \Pi^{\mu\nu}(k) = 0$

Gauge independence!

Ghosts work properly!

Conclusions

- The generating functional of QCD in the Keldysh-Schwinger formalism was constructed.
- The general Slavnov-Taylor identity was derived.
- The ghost Green's function was expressed through the gluon one.
- The computed polarization tensor in the hard loop approximation is automatically transverse.
- **QCD calculations in Keldysh-Schwinger formalism are possible in the Feynman gauge.**