Classical limit of QCD and parton's energy loss

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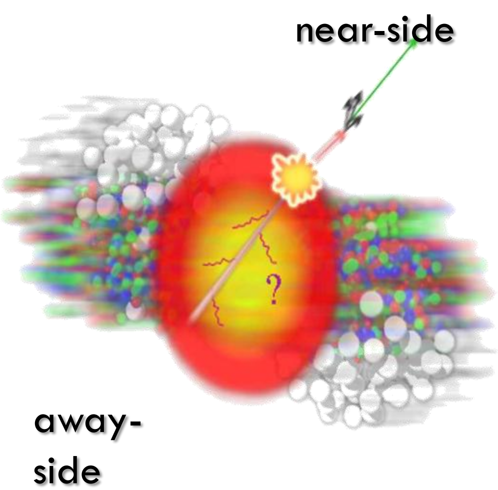
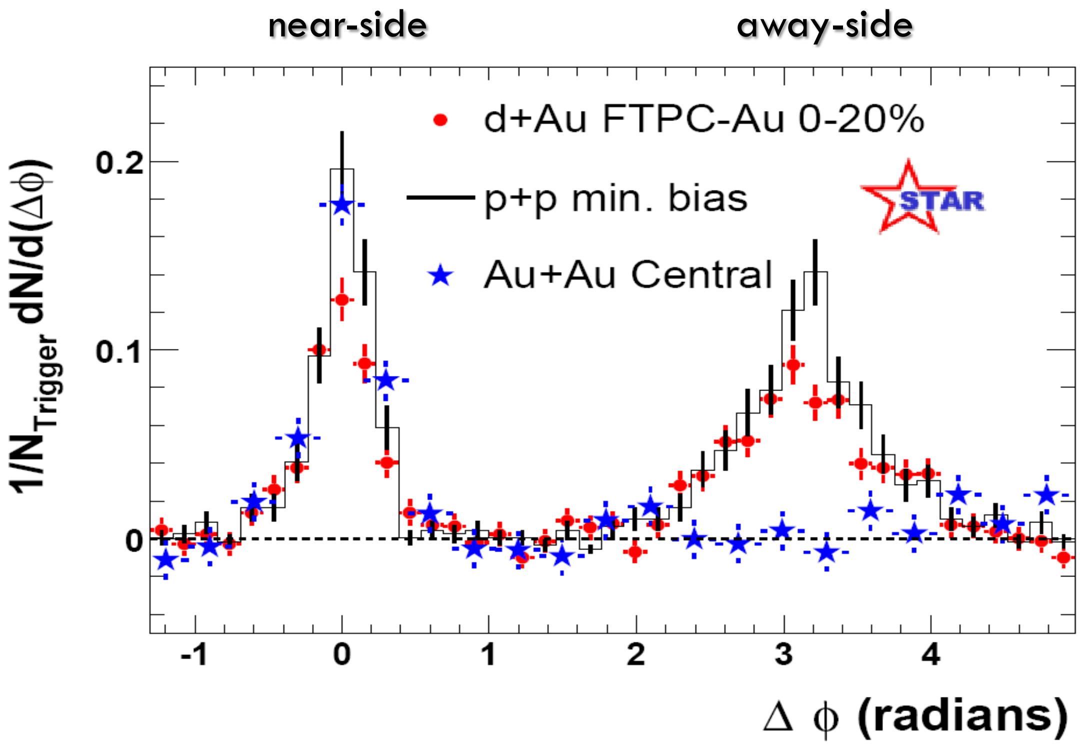
Outline

- Motivation
- Classical approximation
- General energy-loss formula
- Stable system
- Unstable system
  - Prolate system
- Conclusions

Motivation

- Jet quenching

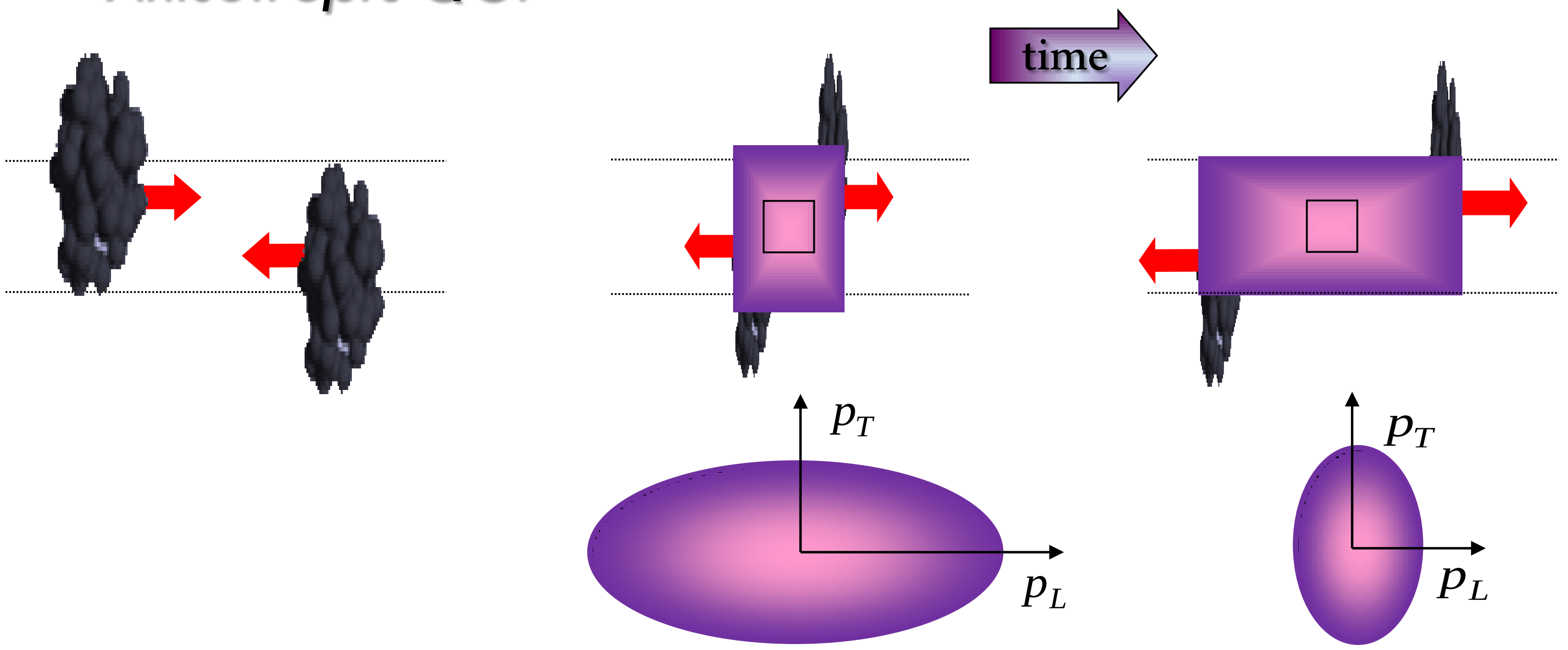




Away-side jet is suppressed in central collisions

Motivation

- *Anisotropic QGP*



Anisotropic QGP is unstable due to magnetic plasma modes

Energy loss in ustable plasma

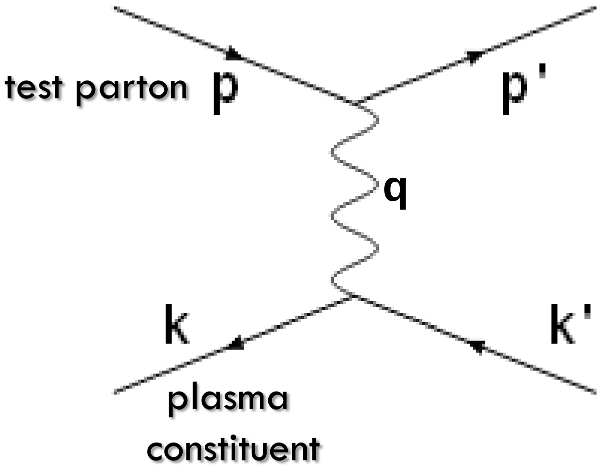
How to calculate energy loss in unstable plasma ?

- May we use classical aproximation

YES !

Classical approximation to $\frac{dE}{dx}$

Scattering due to one-gluon exchange



$$\frac{d\sigma}{dt} \sim \frac{1}{q^4} \sim \frac{1}{q_{\min}^4}$$

$$q_{\min} \sim m_D \sim gT$$

When may we treat the process classically ?

Classical approximation to $\frac{dE}{dx}$

- Classicality of the test particle

$$\lambda << d \sim T^{-1} \quad \Rightarrow \quad p >> T$$

$$\lambda \sim \frac{1}{p}, \quad d \sim n^{-1/3}, \quad n \sim T^{3}$$

λ – wavelength of the test particle
d – distance between particles in the plasma
T – temperature of the plasma
n – density of plasma particles
p – momentum of the test particle
q – momentum of exchanged gluon

- Classicality of the gluon field

$$T >> q$$

Classical approximation to $\frac{dE}{dx}$

$$p >> T >> q$$

$$\frac{d\sigma}{dt} \sim \frac{1}{q_{\min}^4} \sim \frac{1}{g^2 T^2}$$

$$p >> T >> gT$$

$$g << 1$$

In weakly coupled QGP (g<<1) dominant contribution to dE/dx of high-energy parton is classical!

Fast parton in an quark-gluon plasma

Wong's equations of motion

$$\frac{dx^{\mu}(\tau)}{d\tau} = u^{\mu}(\tau),$$

$$\frac{dp^{\mu}(\tau)}{d\tau} = gQ^{a}(\tau)F_{a}{}^{\mu\nu}(x(\tau))u_{\nu}(\tau),$$

$$\frac{dQ_{a}(\tau)}{d\tau} = -gf^{abc}u_{\mu}(\tau)A_{b}^{\mu}(x(\tau))Q_{c}(\tau),$$

First two equations are well known from electrodynamics,
but how to obtain the third one ?

Current conservation in QCD

Covariant current conservation in QCD:

$$D_\mu j^\mu = (\partial_\mu \delta^{ac} + g f^{abc} A_\mu^b) j_c^\mu = 0 \quad \rightarrow \quad Q_a(t) = ?$$

$$j_a^0(t,\mathbf{r}) = gQ_a(t)\delta^{(3)}(\mathbf{r} - \mathbf{r}(t))$$

$$\mathbf{j}_a(t,\mathbf{r}) = gQ_a(t)\mathbf{v}(t)\delta^{(3)}(\mathbf{r} - \mathbf{r}(t))$$

$$\frac{d}{dt}Q_a(t) = -gf^{abc}A_b^\mu(x(t))Q_c(t)\frac{p^\mu}{E_p},$$

$$t \rightarrow \tau$$

proper time

$$\frac{dQ_a(\tau)}{d\tau} = -gf^{abc}u_\mu(\tau)A_b^\mu(x(\tau))Q_c(\tau)$$

Fast parton in an quark-gluon plasma

Wong's equations of motion (Hard Loop Approximation)

$$\frac{dx^\mu(\tau)}{d\tau} = u^\mu(\tau),$$

$$\frac{dp^\mu(\tau)}{d\tau} = gQ^a(\tau)F_a{}^{\mu\nu}(x(\tau))u_\nu(\tau),$$

$$\frac{dQ_a(\tau)}{d\tau} = -gf^{abc}u_\mu(\tau)A_b^\mu(x(\tau))Q_c(\tau),$$

Simplifications

Gauge condition: $u_\mu(\tau)\,A_b^\mu\big(x(\tau)\big) = 0 \quad \Rightarrow \quad Q_a(\tau) = \mathrm{const}$

Parton travels with a constant velocity: $u_\mu = (\gamma\tau, \gamma\mathbf{v}) = \mathrm{const}$

Parton's energy loss

The classical formula of the particle's energy loss per unit time

$$\frac{dE(t)}{dt} = gQ_a \mathbf{E}_a(t,\mathbf{r}) \cdot \mathbf{v}$$

$$\frac{dE(t)}{dt} = \int d^3r \;\; \mathbf{E}_a(t,\mathbf{r}) \cdot \mathbf{j}_a(t,\mathbf{r})$$ gauge invariant form

$\mathbf{j}_a(t,\mathbf{r}) = gQ^a \; \mathbf{v}\delta(\mathbf{r} - \mathbf{v}t)$ - current generated by the test parton

$\mathbf{E}_a(t,\mathbf{r}) = ?$ - chromoelectric field induced in the plasma

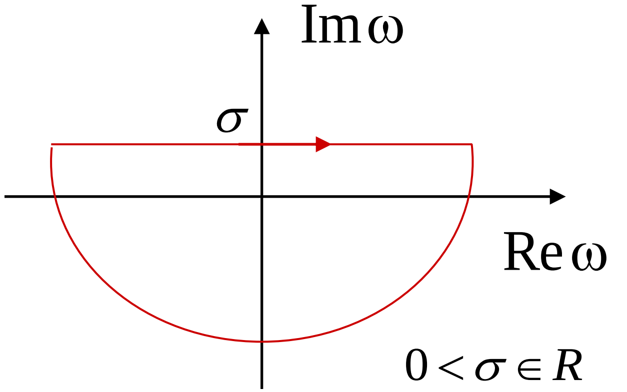
Initial value problem

$$\mathbf{E}(t=0,\mathbf{r}) = \mathbf{E}_0(\mathbf{r}), \quad \mathbf{B}(t=0,\mathbf{r}) = \mathbf{B}_0(\mathbf{r})$$

One-sided Fourier transformation

$$f(\omega,\mathbf{k}) = \int_0^\infty dt \int d^3r\, e^{i(\omega t - \mathbf{kr})} f(t,\mathbf{r})$$

$$f(t,\mathbf{r}) = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega t - \mathbf{kr})} f(\omega,\mathbf{k})$$



$0 < \sigma \in R$

Transformed equation

Energy loss per unit time

$$\frac{dE(t)}{dt} = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int_{-\infty+i\sigma'}^{\infty+i\sigma'} \frac{d\omega'}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega+\omega')t} \mathbf{E}_a(\omega,\mathbf{k}) \cdot \mathbf{j}_a(\omega',-\mathbf{k})$$

?

Linearised Yang-Mills (Maxwell) equations (Hard Loop Approximation)

$$i\mathbf{k}\cdot\mathbf{E}(\omega,\mathbf{k}) = \rho(\omega,\mathbf{k}), \qquad i\mathbf{k}\cdot\mathbf{B}(\omega,\mathbf{k}) = 0,$$

$$i\mathbf{k}\times\mathbf{E}(\omega,\mathbf{k}) = i\omega\mathbf{B}(\omega,\mathbf{k}) + \mathbf{B}_0(\mathbf{k}),$$

$$i\mathbf{k}\times\mathbf{B}(\omega,\mathbf{k}) = \mathbf{j}(\omega,\mathbf{k}) - i\omega\mathbf{E}(\omega,\mathbf{k}) - \mathbf{D}_0(\mathbf{k})$$

$$\begin{cases} \mathbf{D}(\omega,\mathbf{k}) = \hat{\varepsilon}(\omega,\mathbf{k})\mathbf{E}(\omega,\mathbf{k}) \\ \varepsilon^{ij}(k) = \delta^{ij} + \dfrac{g^2}{2\omega} \displaystyle\int \frac{d^3p}{(2\pi)^3} \frac{v^i}{\omega - \mathbf{k}\cdot\mathbf{v} + i0^+} \frac{\partial f(\mathbf{p})}{\partial p^k} \left[\left(1 - \frac{\mathbf{k}\cdot\mathbf{v}}{\omega}\right) \partial^{kj} + \frac{k^k v^j}{\omega} \right] \end{cases}$$

General energy loss formula

$$\frac{dE(t)}{dt} = \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi} \int_{-\infty+i\sigma'}^{\infty+i\sigma'} \frac{d\omega'}{2\pi} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega+\omega')t} \mathbf{E}_a(\omega,\mathbf{k}) \cdot \mathbf{j}_a(\omega',-\mathbf{k})$$

- from Maxwell equations

$$E^j(\omega,\mathbf{k}) = -i\left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k})\left[\omega j^i(\omega,\mathbf{k}) + k \times \mathbf{B}_0^i(\omega,\mathbf{k}) - \omega \mathbf{D}_0^i(\omega,\mathbf{k})\right]$$

- transformed current

$$\mathbf{j}_a(t,\mathbf{r}) = gQ^a \, \mathbf{v}\delta(\mathbf{r}-\mathbf{v}t) \quad \Rightarrow \quad \mathbf{j}_a(\omega,\mathbf{k}) = \frac{igQ^a\mathbf{v}}{\omega - \mathbf{kv}}$$

$$\frac{dE(t)}{dt} = gQ^a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t} \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) \left[\frac{i\omega gQ^a v^j}{\omega-\bar{\omega}} + \varepsilon^{jkl} k^k B_{0a}^l(\mathbf{k}) - \omega D_{0a}^j(\mathbf{k})\right]$$

General energy loss formula

$$\frac{dE(t)}{dt} = gQ^a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t} \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k}) \left[\frac{i\omega g Q^a v^j}{\omega-\bar{\omega}} + \varepsilon^{jkl} k^k B^l_{0a}(\mathbf{k}) - \omega D^j_{0a}(\mathbf{k}) \right]$$

$$\Sigma^{ij}(\omega,\mathbf{k}) = -\mathbf{k}^2 \delta^{ij} + k^i k^j + \omega^2 \varepsilon^{ij}(\omega,\mathbf{k})$$

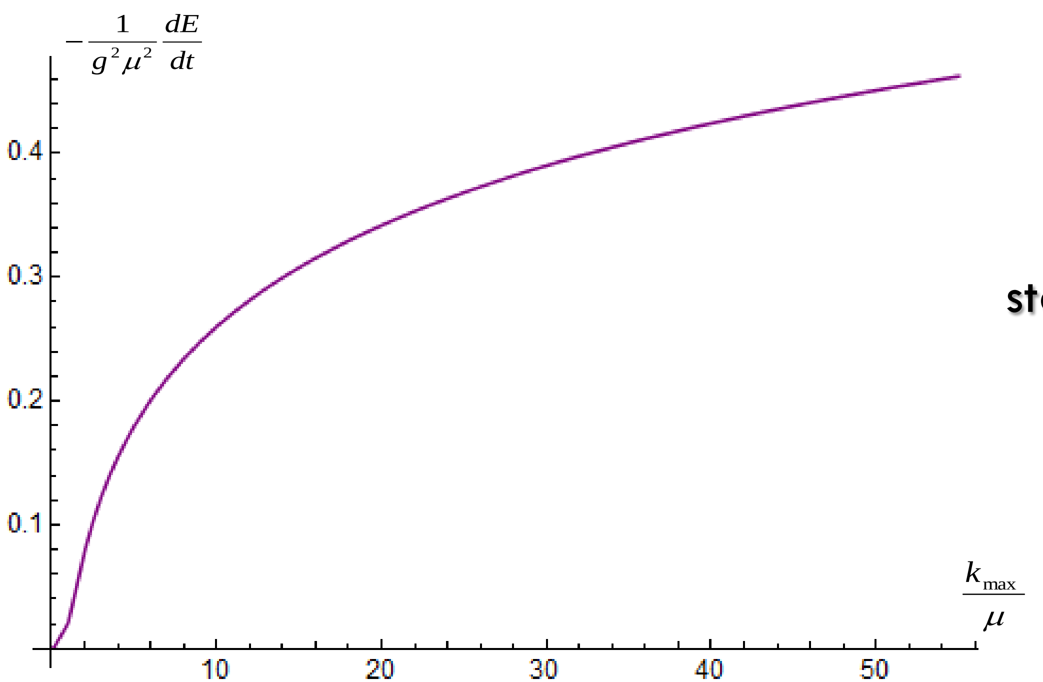
The dispersion equation

$$\det\left[\Sigma(\omega,\mathbf{k})\right] = 0$$

$\omega(\mathbf{k})$ - collective mode in the plasma system

Energy loss in equilibrum plasma

$$\frac{dE(t)}{dt} = -ig^2 C_R \int \frac{d^3k}{(2\pi)^3} \frac{\overline{\omega}}{\mathbf{k}^2} \left[\frac{1}{\varepsilon_L(\overline{\omega},\mathbf{k})} + \frac{\mathbf{k}^2 v^2 - \overline{\omega}^2}{\overline{\omega}^2 \varepsilon_T(\overline{\omega},\mathbf{k}) - \mathbf{k}^2} \right]$$



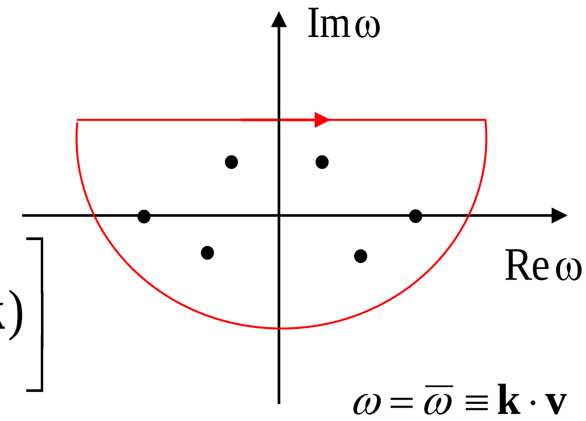
standard result is reproduced

E. Braaten and M. H. Thoma, Phys. Rev. D **44**, 2625 (1991)

Energy loss in unstable system

$$\frac{dE(t)}{dt} = gQ^a v^i \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t} \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k})$$

$$\times\left[\frac{i\omega g Q^a v^j}{\omega-\bar{\omega}} + \varepsilon^{jkl} k^k B^l_{0a}(\mathbf{k}) - \omega D^j_{0a}(\mathbf{k})\right]$$



$$D^i_{0a}(\mathbf{k}) = -gQ^a \bar{\omega} \varepsilon^{ij}(\bar{\omega},\mathbf{k})(\Sigma^{-1})^{jk}(\bar{\omega},\mathbf{k}) v^k$$

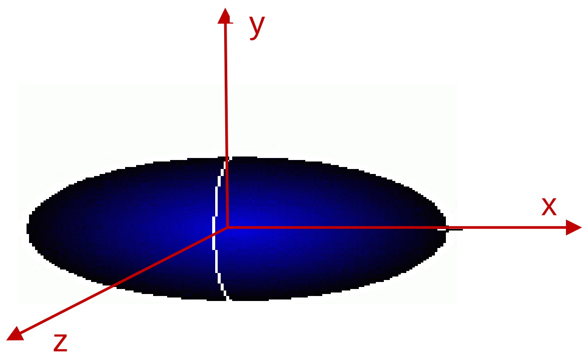
$$B^i_{0a}(\mathbf{k}) = -gQ^a \bar{\omega} \varepsilon^{ijk} k^j (\Sigma^{-1})^{kl}(\bar{\omega},\mathbf{k}) v^l$$

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\bar{\omega})t} \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k})$$

$$\times\left[\frac{\omega\delta^{jl}}{\omega-\bar{\omega}} - \left(k^j k^k - \mathbf{k}^2\delta^{jk}\right)\left(\Sigma^{-1}\right)^{kl}(\bar{\omega},\mathbf{k}) + \omega\bar{\omega}\varepsilon^{jk}(\bar{\omega},\mathbf{k})\left(\Sigma^{-1}\right)^{kl}(\bar{\omega},\mathbf{k})\right]$$

Prolate system

$$f(\mathbf{p}) \sim \delta(p_x)\delta(p_y)$$



n - determines anisotropy: $\mathbf{n} \equiv (0,0,1)$

Inverse propagator: $\Sigma^{ij}(\omega,\mathbf{k}) = -\mathbf{k}^2\delta^{ij} + k^i k^j + \omega^2\varepsilon^{ij}(\omega,\mathbf{k})$

Problem !

How to inverse the matrix Σ ?

Energy loss in prolate system

$$\frac{dE(t)}{dt} = ig^2 C_R v^i v^l \int_{-\infty+i\sigma}^{\infty+i\sigma} \frac{d\omega}{2\pi i} \int \frac{d^3k}{(2\pi)^3} e^{-i(\omega-\overline{\omega})t} \left(\Sigma^{-1}\right)^{ij}(\omega,\mathbf{k})$$

$$\times\left[\frac{\omega\delta^{jl}}{\omega-\overline{\omega}} - \left(k^j k^k - \mathbf{k}^2\delta^{jk}\right)\left(\Sigma^{-1}\right)^{kl}(\overline{\omega},\mathbf{k}) + \omega\overline{\omega}\varepsilon^{jk}(\overline{\omega},\mathbf{k})\left(\Sigma^{-1}\right)^{kl}(\overline{\omega},\mathbf{k})\right]$$

Inversion of the matrix Σ which depends on **k** and **n**

$$\Sigma = \alpha A + \beta B + \gamma C + \delta D$$

basis of matrices

$$A^{ij} = \delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}, \quad B^{ij} = \frac{k^i k^j}{\mathbf{k}^2}$$

$$C^{ij} = \frac{n_T^i n_T^j}{\mathbf{n}_T^2}, \quad D^{ij} = n_T^i k^j + k^i n_T^j$$

$$n_T^i = \left(\delta^{ij} - \frac{k^i k^j}{\mathbf{k}^2}\right) n^j$$

$$\Sigma^{-1} = \overline{\alpha} A + \overline{\beta} B + \overline{\gamma} C + \overline{\delta} D$$

$$\Sigma\Sigma^{-1} = \mathbf{1} \quad \Rightarrow \quad \overline{\alpha}, \overline{\beta}, \overline{\gamma}, \overline{\delta}$$

P. Romatschke and M. Srickland, Phys. Rev. D **68**, 036004 (2003)

Collectives modes in prolate system

Collective modes

$$\Sigma^{ij}(\omega,\mathbf{k}) = \left(\omega^2 - \mu^2 - \mathbf{k}^2\right)\delta^{ij} + k^i k^j - \frac{\mu^2 \mathbf{k}\cdot\mathbf{n}}{\omega^2 - (\mathbf{k}\cdot\mathbf{n})^2}\left(k^i n^j + n^i k^j\right) - \frac{\mu^2\left(\omega^2 + (\mathbf{k}\cdot\mathbf{n})^2\right)\left(\mathbf{k}^2 - \omega^2\right)}{\left(\omega^2 - (\mathbf{k}\cdot\mathbf{n})^2\right)^2} n^i n^j$$

$$\det\left[\Sigma^{ij}(\omega,\mathbf{k})\right] = 0$$

Spectrum of collective modes

$$\omega_1^2(k) = \mu^2 + \mathbf{k}^2$$

$$\omega_2^2(k) = \mu^2 + (\mathbf{k}\cdot\mathbf{n})^2$$

$$\omega_\pm^2 = \frac{1}{2}\left(\mathbf{k}^2 + (\mathbf{k}\cdot\mathbf{n})^2 \pm \sqrt{\mathbf{k}^4 + (\mathbf{k}\cdot\mathbf{n})^4 + 4\mu^2\mathbf{k}^2 - 4\mu^2(\mathbf{k}\cdot\mathbf{n})^2 - 2\mathbf{k}^2(\mathbf{k}\cdot\mathbf{n})^2}\right)$$

Energy loss in prolate system

There are 10 contributions corresponding to

$$\omega = \pm\omega_1(\mathbf{k}), \pm\omega_2(\mathbf{k}), \pm\omega_+(\mathbf{k}), \pm\omega_-(\mathbf{k}), \overline{\omega} = \mathbf{k}\cdot\mathbf{v}, 0$$

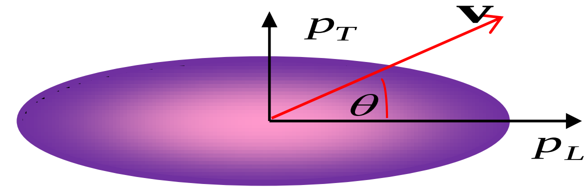
Only one dimensional parameter: $\mu^2 = g^2 \int \frac{d^3p}{(2\pi)^3} \frac{f(\mathbf{p})}{|\mathbf{p}|}$

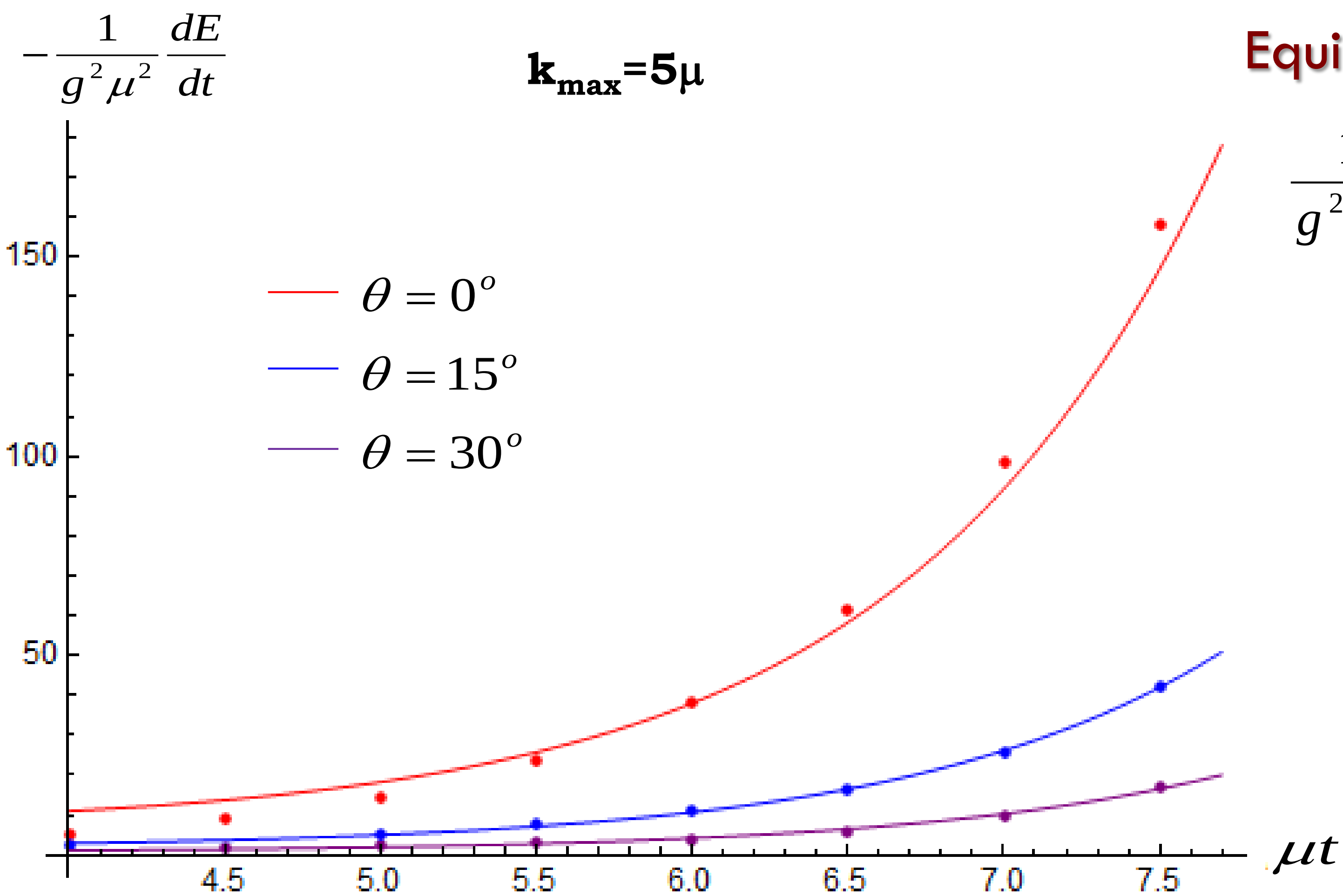
Remaining parameters: $g = 1, \quad |\mathbf{v}| = 1, \quad |\mathbf{n}| = 1, \quad C_R = 3$

The integral over **k** is performed numerically for

$$-k_{\max} < k_L < k_{\max}, \quad 0 < k_T < k_{\max}$$

Numerical analysis





Equilibrium value

$$\frac{1}{g^2\mu^2}\frac{dE}{dt} \approx 0.2$$

Conclusions

- The formalism to compute the energy loss in unstable QGP is developed.
- The standard equilibrium result is reproduced.
- The energy loss in unstable system is strongly time and directionally dependent.
- dE/dx in an unstable QGP is much bigger than in equilibrium on.