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Dynamics of effective gluons

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In Hamiltonians calculated in renormalization group procedure for effective particles (RGPEP) in light-front QCD, interaction terms vary with the scale s that has interpretation of the size of effective particles. Asymptotic freedom appears when s tends to zero. The rate of change of the three-gluon vertex with s depends in a finite way on the regularization at small Bjorken x. A class of regularizations is found to lead to the familiar results. In Hamiltonians with large size s, vertex form factors suppress interactions with large changes of kinetic energy and direct couplings of low-energy constituents to high-energy components in the bound-state dynamics are removed. The RGPEP can be formulated in a non-perturbative way for the purpose of explanation of parton and constituent models of hadrons.

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Outline

1. Start with the Lagrangian density,
2. to construct the Hamiltonian
3. using the renormalization group procedure for effective particles (RGPEP)
4. and producing the dynamics of effective quarks and gluons,
5. with which you may attempt solving the Schrödinger equation in QCD.

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$$\boxed{\mathcal{L} = -\frac{1}{2}\,\mathrm{tr}\,F^{\mu\nu}F_{\mu\nu}}$$

$\mathcal{H} = ?$ $\quad H|\psi\rangle = E|\psi\rangle$, $\quad |\psi\rangle = \text{Hadron}$, $\quad |0\rangle = ?$ $\quad$ BPTh

$p^+ = p^0 + p^3 \qquad p^\perp = (p^1, p^2) \qquad p^-$

$x^- = x^0 - x^3 \qquad x^\perp = (x^1, x^2) \qquad x^+ = x^0 + x^3 \qquad P^- = \frac{1}{2}\int dx^- d^2x^\perp\, \mathcal{H}|_{x^+=0}$

$$\mathcal{H} = \mathcal{H}_{A^2} + \mathcal{H}_{A^3} + \mathcal{H}_{A^4} + \mathcal{H}_{[\partial AA]^2}$$

$$\mathcal{H}_{A^2} = -\frac{1}{2}A^\perp(\partial^\perp)^2 A^\perp$$

$$\mathcal{H}_{A^3} = g\,i\,\partial_\alpha A^a_\beta [A^\alpha, A^\beta]^a$$

This talk's focus:

$$\mathcal{H}_{A^4} = -\frac{1}{4}g^2[A_\alpha, A_\beta]^a[A^\alpha, A^\beta]^a$$

$$\mathcal{H}_{[\partial AA]^2} = \frac{1}{2}g^2[i\partial^+ A^\perp, A^\perp]^a \frac{1}{(i\partial^+)^2}[i\partial^+ A^\perp, A^\perp]^a$$

$$A^\mu = \sum_{\sigma c}\int [k]\left[t^c \varepsilon^\mu_{k\sigma} a_{k\sigma c} e^{-ikx} + t^c \varepsilon^{\mu *}_{k\sigma} a^\dagger_{k\sigma c} e^{ikx}\right]_{x^+=0}$$

complexity

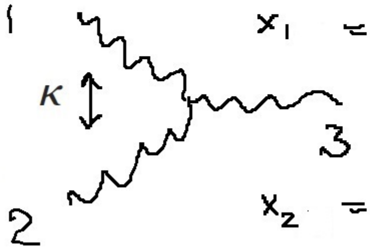
$$H_{A^3} = \frac{1}{2} \int d^3x \; g\, i \partial_\alpha A^a_\beta [A^\alpha, A^\beta]^a \longleftarrow$$

This talk's focus:

$$A^\mu = \sum_{\sigma c} \int [k][t^c \varepsilon^\mu_{k\sigma} a_{k\sigma c} e^{-ikx} + t^c \varepsilon^{\mu *}_{k\sigma} a^\dagger_{k\sigma c} e^{ikx}]$$

$$H_{A^3} = \sum_{123} \int [123] \tilde{\delta}(p^\dagger - p) \tilde{r}_{\Delta\delta}(3,1)[g Y_{123} a^\dagger_1 a^\dagger_2 a_3 + g Y^*_{123} a^\dagger_3 a_2 a_1]$$

$$Y_{123} = i f^{c_1 c_2 c_3} \left[\varepsilon^*_1 \varepsilon^*_2 \cdot \varepsilon_3 \kappa - \varepsilon^*_1 \varepsilon_3 \cdot \varepsilon^*_2 \kappa \frac{1}{x_2} - \varepsilon^*_2 \varepsilon_3 \cdot \varepsilon^*_1 \kappa \frac{1}{x_1} \right]$$



$x_1 = \frac{p_1^+}{p_3^+}$

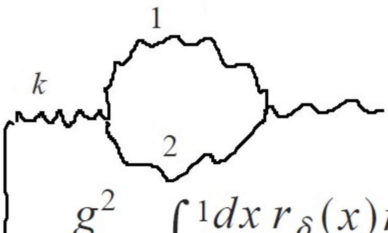
$x_2 = \frac{p_2^+}{p_3^+}$

$\varepsilon_3 \kappa = \varepsilon_3^\perp \cdot \kappa^\perp$

$\varepsilon_1^* \varepsilon_2^* = \varepsilon_1^{*\perp} \cdot \varepsilon_2^{*\perp}$

$\kappa^\perp = x_2 p_1^\perp - x_1 p_2^\perp$

$\tilde{r}_{\Delta\delta}(3,1)$ = regularization



$$\Sigma_{12}|Y_{12k}|^2 = N_c \kappa^2 [1 + 1/x^2 + 1/(1-x)^2]$$

$$= \kappa^2 P(x)/[2x(1-x)]$$

$$= \frac{g^2}{16\pi^3} \int_0^1 \frac{dx\, r_\delta(x) r_\delta(1-x)}{x(1-x)} \int d^2\kappa^\perp \; \frac{N_c \kappa^2 [1 + 1/x^2 + 1/(1-x)^2]}{\mathcal{M}^2} r_\Delta(\kappa)$$

$$\infty$$

$$\mathcal{M}^2 = \frac{\kappa^{\perp 2}}{x(1-x)}$$

three choices to discuss

$$r_\delta(x) = x/(x+\delta)$$

$$r_\Delta(\kappa) = \exp(-\kappa^2/\Delta^2)$$

$$r_\delta(x) = \theta(x-\delta) \qquad \delta \to 0$$

$$\Delta \to \infty$$

$$r_\delta(x) = x^\delta \theta(x-\epsilon) \qquad \delta \gg \epsilon$$

$$\boxed{H\,|\psi\rangle = E\,|\psi\rangle}$$

- badly divergent integrals
- infinitely many components
- with increasingly many quanta

$a = a_{s=0} = a_0$

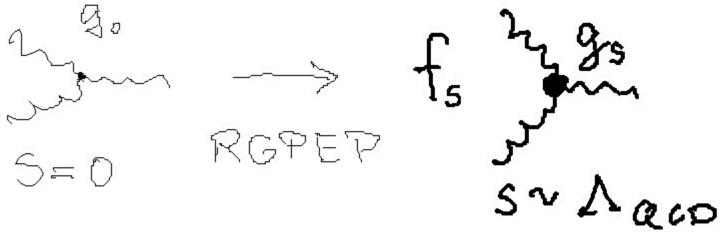
s = size of an effective gluon

$q_0 = a_0$ or $a_0^\dagger$

$\lambda = \frac{1}{s}$ width of vertex form factors in effective QCD

$$q_s \;=\; \mathcal{U}_s\, q_0\, \mathcal{U}_s^\dagger$$

$$\boxed{\mathcal{H}_s(q_s) \;=\; \mathcal{H}_0(q_0)}$$



$$\mathcal{H}_s = \sum_{n=2}^{\infty} \sum_{i_1, i_2, \ldots, i_n} c_s(i_1, \ldots, i_n)\, q^\dagger_{i_1} \ldots q_{i_n}$$

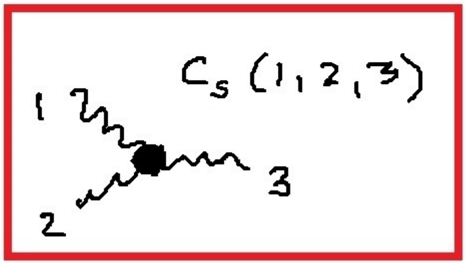
$$\mathcal{H}_s(q_0) = \mathcal{U}_s^\dagger \, \mathcal{H}_0(q_0) \, \mathcal{U}_s$$

$$\mathcal{G}_s = -\mathcal{U}_s^\dagger \mathcal{U}_s'$$

generator of the RGPEP

$$\mathcal{H}_s'(q_0) = \left[-\mathcal{U}_s^\dagger \mathcal{U}_s', \mathcal{H}_s(q_0) \right]$$

Make a choice for $\mathcal{G}_s$ and you obtain $c_s(i_1, \ldots, i_n)$.



$$H_{A^3 0} = \sum_{123} \int [123] \tilde{\delta}\, r_{\Delta\delta}(3,1) [g Y_{123}\, a^\dagger_{01} a^\dagger_{02} a_{03} + g Y^*_{123} a^\dagger_{03}\, a_{02}\, a_{01}]$$

V

$$H_{A^3 s} = \sum_{123} \int [123] \tilde{\delta}\, f_s [V_{s12}\, Y_{12} + V_{s13} Y_{13} + V_{s23}\, Y_{23} + V_{s4}\, Y_4] \cdot$$

$$\cdot\, a^\dagger_{s1} a^\dagger_{s2} a_{s3} + \text{h.c.}$$

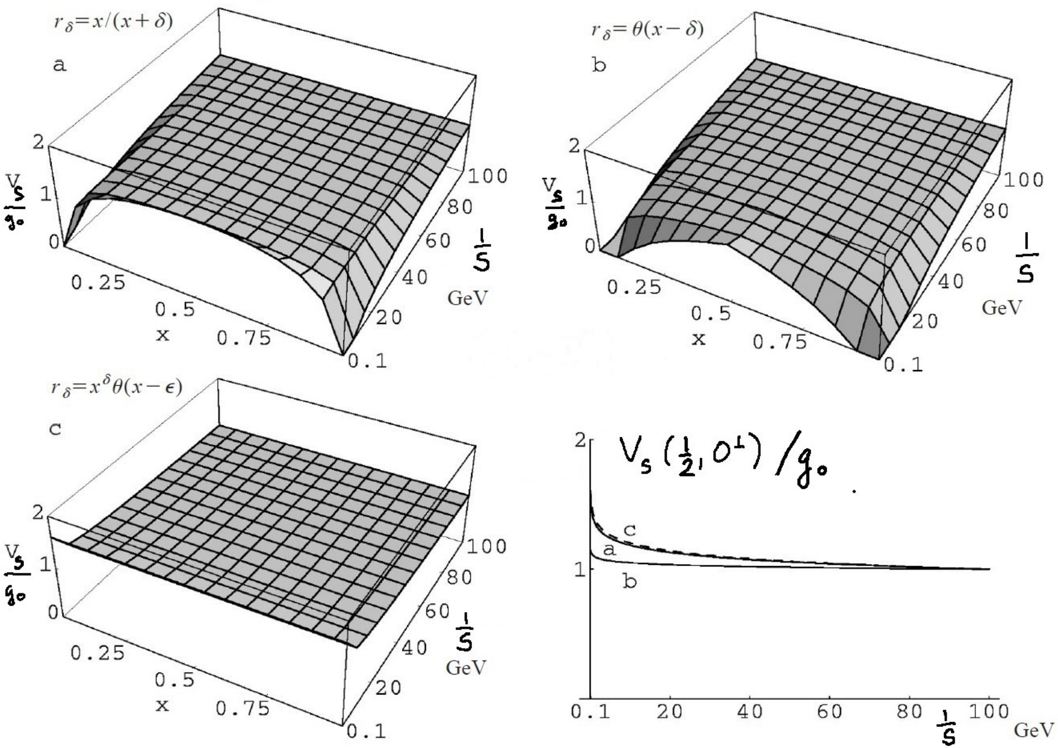
$$Y_{123} = i f^{c_1 c_2 c_3} \left[\underbrace{\varepsilon_1^* \varepsilon_2^* \cdot \varepsilon_3 \kappa}_{Y_{12}} - \underbrace{\varepsilon_1^* \varepsilon_3 \cdot \varepsilon_2^* \kappa \frac{1}{x_{2/3}}}_{Y_{13}} - \underbrace{\varepsilon_2^* \varepsilon_3 \cdot \varepsilon_1^* \kappa \frac{1}{x_{1/3}}}_{Y_{23}} \right]$$

$$Y_4 = i f^{c_1 c_2 c_3} \varepsilon_1^* \kappa \cdot \varepsilon_2^* \kappa \cdot \varepsilon_3 \kappa / \kappa^2 \qquad ?$$

$f_s = \exp\{-[s^2 \kappa^2 / x(1-x)]^2\}$ vertex form factor of RGPEP

$$V_{s\,ij}(x_1, \kappa^2) \qquad g_{s_0} \equiv V_{s_0 ij}\left(\tfrac{1}{2}, 0\right) = g_0 \longrightarrow V_s(x, 0^\perp)$$

$$\alpha_0 = g_0^2/(4\pi) = 0.1 \quad \text{for} \quad \frac{1}{s_0} = 100\,\text{GeV}$$



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Conclusion

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- One can attempt to tame QCD using the RGPEP,
- evaluating from it dynamics of effective quarks and gluons,
- and solving the Schrödinger equation using the latter,
- which is not as violent and multi-scale as the former.

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