Double parton distributions

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Outline

- Motivation
- Basic definitions
- Evolution equations
- Sum rules
- Initial conditions for evolution
- Application to double W boson production
- Summary

Motivation

- Double parton distribution functions (DPDF) provide new information about structure of hadrons.
- DPDF are used in the description of double parton scattering (DPS).
- DPS becomes increasingly important with rising energy - LHC.
- DPS as background to many processes - Higgs production.

DPS studies

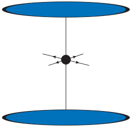
- Theoretical:

  Snigirev, Korotkikh, Furmanski, Bartels, Lipatov, Dokshitzer, Frankfurt, Strikman, Ryskin, Gaunt, Stirling, Diehl, Ceccopieri, Treleani, Manohar, Waalewijn ...

- Experimental:
  - Axial Field Spectrometer Collaboration (1987) - pp scattering, $\sqrt{s} = 63\ \mathrm{GeV}$, double dijets: $jj + jj$
  - CDF Collaboration (1997) - $p\bar{p}$, $\sqrt{s} = 1.8\ \mathrm{TeV}$, double dijets: $jj + jj$
  - D0 Collaboration (2010) - $p\bar{p}$, $\sqrt{s} = 1.96\ \mathrm{TeV}$, $\gamma + 3$ jets: $\gamma j + jj$
  - ATLAS (2013) - pp, $\sqrt{s} = 7$ TeV, production $W + jj$

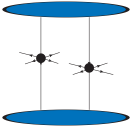
Parton distribution functions

- Single parton scattering



PDF: $D_f(x, Q)$

- Double parton scattering



DPDF: $D_{f_1 f_2}(x_1, x_2, Q_1, Q_2; q)$

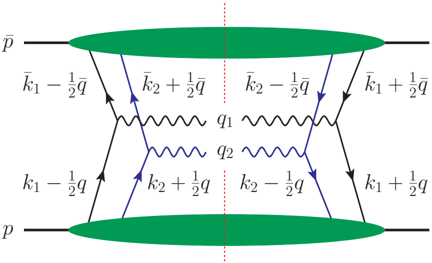
- Sum of partons' momenta cannot exceed total nucleon momentum

$$x_1 + x_2 \leq 1$$

- Two hard scales: Q_1, Q_2 and two flavours: f_1, f_2 (including gluon).

- Transverse momentum flowing in the loop: q.

DPS cross section



- Integration over transverse momenta: $k_1, k_2, \overline{k}_1, \overline{k}_2$ - collinear DPDF.
- DPS cross section in the collinear approximation:

$$\frac{d\sigma}{dx_1\, dx_2\, d\overline{x}_1\, d\overline{x}_2} = \int \frac{d^2q}{(2\pi)^2}\, D_{ij}(x_1, x_2, Q_1, Q_2; q)\ \sigma^1_{i\bar{i}}\ \sigma^2_{j\bar{j}}\ D_{\bar{i}\bar{j}}(\overline{x}_1, \overline{x}_2, Q_1, Q_2; -q)$$

Standard approach to DPS

- Experimental factorization: D0 and $\gamma + 3j$ production

$$\sigma^{\gamma+3j} = \frac{\sigma^{\gamma j}\,\sigma^{jj}}{\sigma_{eff}} \qquad\qquad \sigma_{eff} = 15 \text{ mb}$$

- Theoretical factorization:

$$D_{ij}(x_1, x_2, Q_1, Q_2; q) = D_i(x_1, Q_1)\, D_j(x_2, Q_2)\, F(q^2)$$

- Nonperturbative partonic form-factor

$$F(q^2) = \frac{1}{(1 + q^2/m^2)^4}$$

- Agreement between experiment and theory:

$$\frac{1}{\sigma_{eff}} = \int \frac{d^2q}{(2\pi)^2}\, F^2(q^2) \qquad => \qquad m = 1.5 \text{ GeV}$$

Critical analysis

- PDF evolve through QCD evolution equations (DGLAP):

$$D_f(x, Q_0) \quad \rightarrow \quad D_f(x, Q)$$

- DPDF also evolve through QCD evolution equations ($Q_1 < Q_2$):

$$D_{f_1 f_2}(x_1, x_2, Q_0, Q_0) \;\rightarrow\; D_{f_1 f_2}(x_1, x_2, Q_1, Q_1) \;\rightarrow\; D_{f_1 f_2}(x_1^{fix}, x_2, Q_1, Q_2)$$

- Is factorization ansatz compatible with the new evolution equations?

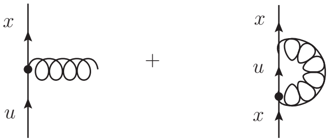
- Analysis with $q = 0$.

QCD evolution equations for single PDF

- General form of evolution equations for single PDF ($t = \ln Q^2$)

$$\partial_t D_f(x,t) = \sum_{f'} \int_0^1 du\, \mathcal{K}_{ff'}(x,u,t)\, D_{f'}(u,t)$$

- The integral kernels describe real and virtual parton emission



$$\mathcal{K}_{ff'}(x,u,t) = \mathcal{K}^R_{ff'}(x,u,t) - \delta(u-x)\,\delta_{ff'}\,\mathcal{K}^V_f(x,t)$$

Evolution equations (cont.)

- The real emission kernels

$$\mathcal{K}^R_{ff'}(x,u,t) = \frac{1}{u}\, P_{ff'}(\frac{x}{u},t)\, \theta(u-x)$$

- Splitting functions

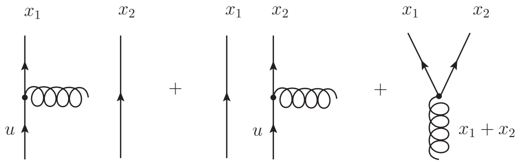
$$P_{ff'}(z,t) = \frac{\alpha_s(t)}{2\pi} P^{(0)}_{ff'}(z) + \frac{\alpha_s^2(t)}{(2\pi)^2} P^{(1)}_{ff'}(z) + \ldots$$

- Well known DGLAP evolution equations for single PDF

$$\partial_t D_f(x,t) = \sum_{f'} \int_x^1 \frac{dz}{z} P_{ff'}(z,t)\, D_{f'}(\frac{x}{z},t) - D_f(x,t) \sum_{f'} \int_0^1 dz\, z\, P_{f'f}(z,t)$$

Evolution equations for DPDF

- Evolution with equal scales: $D_{f_1 f_2}(x_1, x_2, t) = D_{f_1 f_2}(x_1, x_2, Q, Q)$



- Evolution equations with three terms:

$$
\begin{aligned}
\partial_t D_{f_1 f_2}(x_1, x_2, t) &= \sum_{f'} \int_0^{1-x_2} du\, \mathcal{K}_{f_1 f'}(x_1, u, t)\, D_{f' f_2}(u, x_2, t) \\
&+ \sum_{f'} \int_0^{1-x_1} du\, \mathcal{K}_{f_2 f'}(x_2, u, t)\, D_{f_1 f'}(x_1, u, t) \\
&+ \sum_{f'} \mathcal{K}^R_{f' \to f_1 f_2}(x_1, x_1 + x_2, t)\, D_{f'}(x_1 + x_2, t)
\end{aligned}
$$

Splitting term

- The third, splitting term contains single PDF

$$\frac{\alpha_s(Q)}{2\pi} \sum_{f'} \frac{1}{x_1 + x_2} P_{f' \to f_1 f_2}\left(\frac{x_1}{x_1 + x_2}\right) D_{f'}(x_1 + x_2, Q)$$

- Evolution equations for PDF and DPDF need to be solved together.

- Initial conditions for both PDF and DPDF have to be specified:

$$D_f(x, Q_0) \;, \qquad\qquad D_{f_1 f_2}(x_1, x_2, Q_0)$$

Momentum sum rule

- Momentum sum rule for PDF (conserved by DGLAP eqs.)

$$\sum_f \int_0^1 dx\, x\, D_f(x, Q) = 1$$

- By analogy: momentum sum rule for DPDF

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, \frac{D_{f_1 f_2}(x_1, x_2, Q)}{D_{f_2}(x_2, Q)} = (1 - x_2)$$

- Conditional probablility to find parton 1 with known parton 2.

- Momentum sum rule is a relation between DPDF and PDF:

$$\sum_{f_1} \int_0^{1-x_2} dx_1\, x_1\, D_{f_1 f_2}(x_1, x_2, Q) = (1 - x_2)\, D_{f_2}(x_2, Q) \qquad (1)$$

Valence number sum rule

- Valence number sum rule for PDF (N_i= no. of valence quarks)

$$\int_0^1 dx\,\{D_{q_i}(x,Q) - D_{\bar{q}_i}(x,Q)\} = N_i$$

- Valence number sum rule for DPDF

$$\begin{aligned} I_{q_i f_2} &= \int_0^{1-x_2} dx_1\,\{D_{q_i f_2}(x_1,x_2,Q) - D_{\bar{q}_i f_2}(x_1,x_2,Q)\} \\ &= \begin{cases} N_i\,D_{f_2}(x_2,Q) & \text{for } f_2 \neq q_i,\bar{q}_i \\ (N_i - 1)\,D_{f_2}(x_2,Q) & \text{for } f_2 = q_i \\ (N_i + 1)\,D_{f_2}(x_2,Q) & \text{for } f_2 = \bar{q}_i \end{cases} \end{aligned} \tag{2}$$

- Eqs. (1) and (2) are conserved by the evolution equations if imposed on initial conditions at some scale Q_0.

Symmetric initial conditions

- In practice: initial DPDF are built out of the existing single PDF, e.g. (Gaunt, Stirling, Korotkikh, Snigirev)

$$D_{f_1 f_2}(x_1, x_2) = D_{f_1}(x_1)\, D_{f_2}(x_2)\, \frac{(1 - x_1 - x_2)^2}{(1 - x_1)^{2+n_1}(1 - x_2)^{2+n_2}}$$

- Symmetric with respect to parton interchange

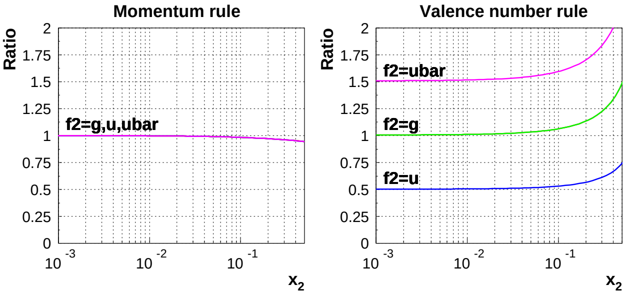
$$D_{f_1 f_2}(x_1, x_2) = D_{f_2 f_1}(x_2, x_1)$$

- Positive definite.

Sum rules with the symmetric input

$$\mathrm{Ratio}_{\mathrm{Mom}}(x_2, f_2, Q) = \frac{L.H.S\ of\ eq.(1)}{R.H.S\ of\ eq.(1)} \qquad (=1\ ?)$$

$$\mathrm{Ratio}_{\mathrm{Val}}(x_2, f_2, Q) = \frac{L.H.S\ of\ eq.(2)}{R.H.S\ of\ eq.(2)} \qquad (=1\ ?)$$



Valence quark number sum rule is violated.

How to *exactly* fulfill the sum rules?

- Asymmetric ansatz to fulfill the momentum sum rule (1):

$$D_{f_1 f_2}(x_1, x_2) \; = \; \frac{1}{1-x_2} D_{f_1}(\frac{x_1}{1-x_2}) \cdot D_{f_2}(x_2)$$

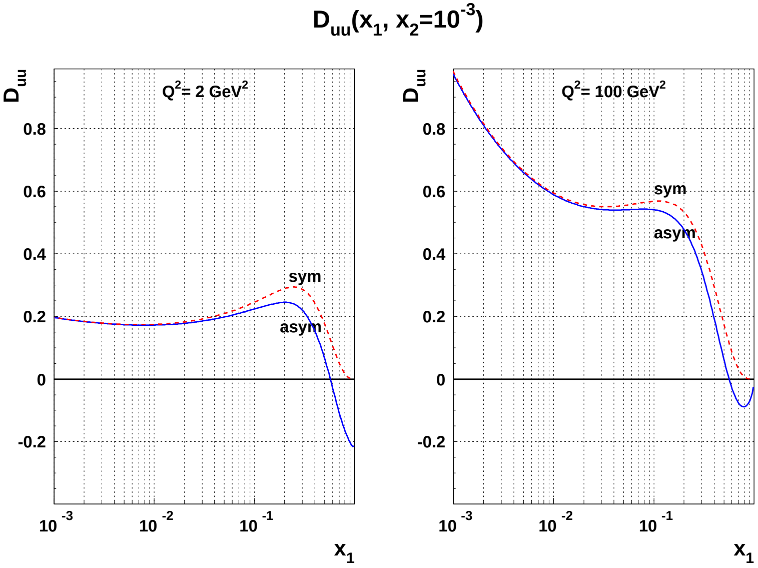
- Corrections for identical quark flavours/antiflavours to obey the valence number sum rule (2):

$$D_{f_i f_i}(x_1, x_2) \; = \; \frac{1}{1-x_2} \left\{ D_{f_1}(\frac{x_1}{1-x_2}) - \frac{1}{2} \right\} D_{f_1}(x_2)$$

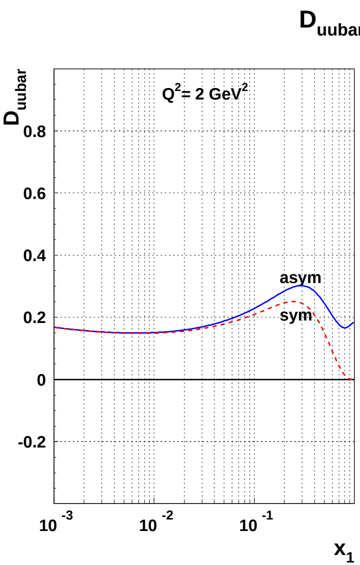
$$D_{f_i \bar{f}_i}(x_1, x_2) \; = \; \frac{1}{1-x_2} \left\{ D_{f_1}(\frac{x_1}{1-x_2}) + \frac{1}{2} \right\} D_{\bar{f}_1}(x_2)$$

- DPDF for identical flavours/antiflavours are not positive definite.
- This is the price for the construction with single PDF !

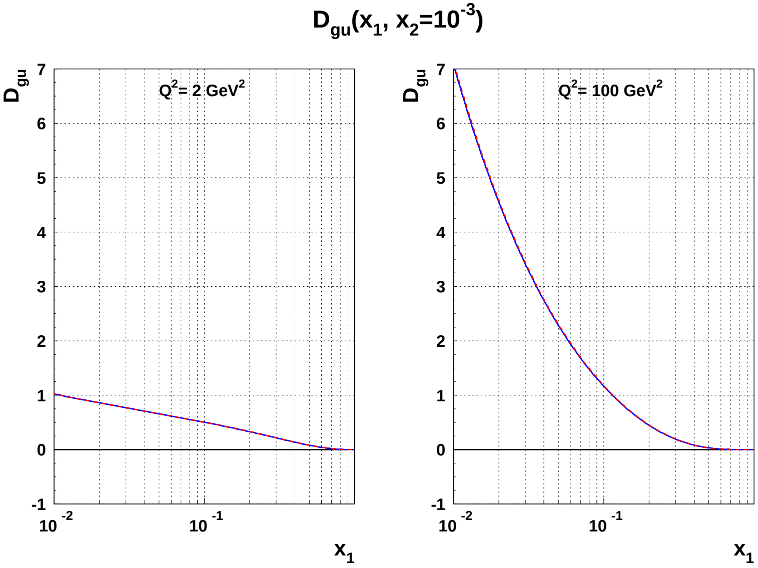
Symmetric vs. asymmetric input for D_{uu}



Symmetric vs. asymmetric input for $D_{u\bar{u}}$



Symmetric vs. asymmetric input for D_{gu}



Summary of DPDF evolution

- If input DPDF are constructed from the known single PDF:

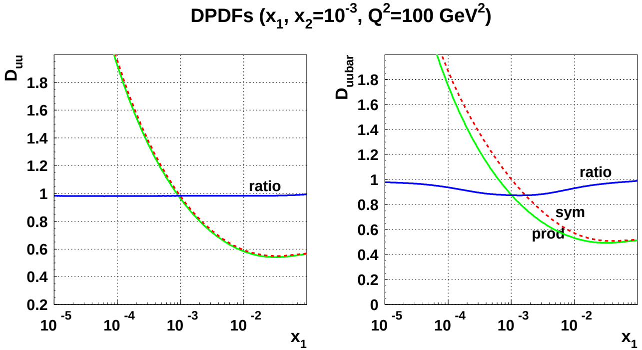
	symmetric input	our asymmetric input
Parton symmetry	yes	no
Positivity	yes	no
Sum rules	no	yes

- Alternative: specify positive initial DPDF and generate initial PDF using sum rules. (Work in progress with Wojtek Broniowski).

- For $x_1, x_2 \to 0$ factorized form is a good approximation:

$$D_{f_1 f_2}(x_1, x_2, Q) \approx D_{f_1}(x_1, Q)\, D_{f_2}(x_2, Q)$$

DPDF factorization at small x

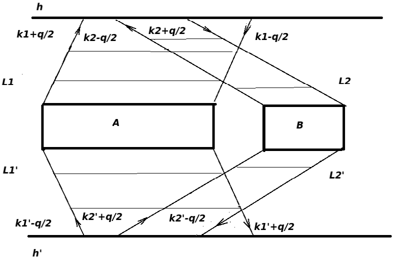


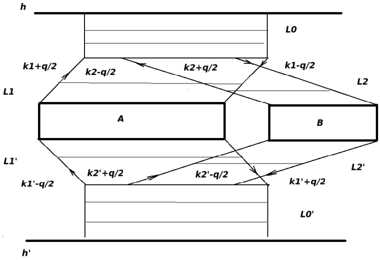
- Splitting term in DPDF evolution equations: $g \to q\bar{q}$ is responsible for 10% violation of factorization near $x_1 \sim 10^{-3}$ because of small-x gluon distribution $g(x)$.

The role of partonic formfactor $F(q^2)$

- Up till now we consider DPDF with $q = 0$. Now $q \neq 0$.
- General solution is the sum of two solutions - for homogeneous and nonhomogenenous evolution equation:

$$D_{f_1 f_2}(x_1, x_2, Q, q) = D^{(1)}_{f_1 f_2}(x_1, x_2, Q) F(q^2) + D^{(2)}_{f_1 f_2}(x_1, x_2, q, Q)$$





(Ryskin, Snigirev 2011)

Particular solution

- $D^{(2)}_{f_1 f_2}$ is a particular solution of the nonhomogeneous equation:

$$D^{(2)}_{f_1 f_2}(x_1, x_2, q, Q) = \int_q^Q dk \frac{\alpha_s(k)}{\pi k} \int_{x_1}^{1-x_2} \frac{dz_1}{z_1} \int_{x_2}^{1-z_1} \frac{dz_2}{z_2}$$

$$\times \; D_{f'}(z_1 + z_2, Q_0, k) \, P_{f' \to f_1' f_2'}\left(\frac{z_1}{z_1 + z_2}\right) D_{f_1}^{f_1'}\left(\frac{x_1}{z_1}, k, Q\right) D_{f_2}^{f_2'}\left(\frac{x_2}{z_2}, k, Q\right)$$

- We sum over splittings at $k \in [q, Q]$.
- No damping formfactor for pointlike parton.

Double parton scattering cross section

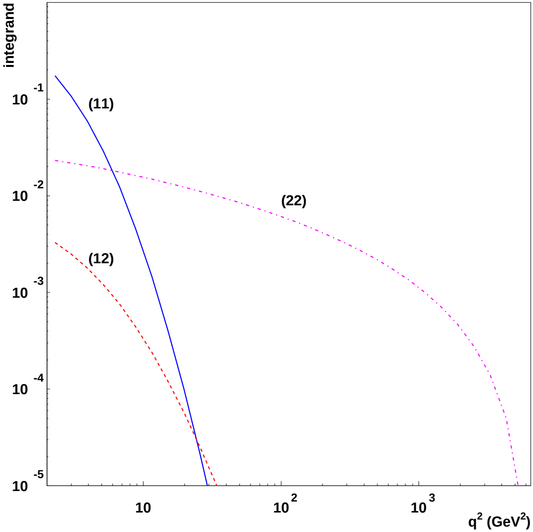
- DPS cross section written with the help of the two DPDF contributions:

$$\sigma = \int \frac{d^2q}{2\pi} \left\{ D_{ij}^{(1)} F(q^2) + D_{ij}^{(2)}(q) \right\} \sigma_{im}^1 \, \sigma_{jn}^2 \left\{ D_{mn}^{(1)} F(q^2) + D_{mn}^{(2)}(q) \right\}$$

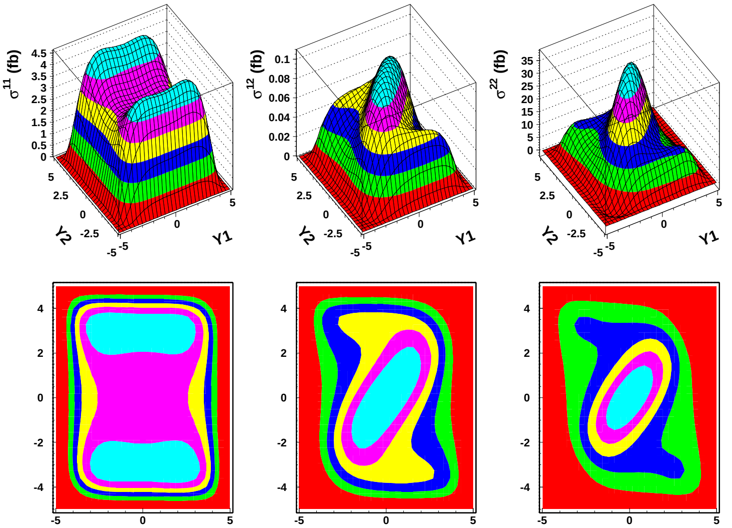
$$= \sigma^{(11)} + \sigma^{(12)} + \sigma^{(22)}$$

- Tested in W^+W^- DPS production.

Formfactor suppression



W^+W^- production from DPS



Summary

- For large $Q = M_W$ splitting mechanism dominates DPS.
- Standard DPS part is suppressed by partonic formfactor $F^2(q^2)$.
- The factorized formula for DPS cross section is no longer valid.