AdS/CFT, hydrodynamics and beyond

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M. Heller, RJ, P. Witaszczyk, M. Spaliński, work in progress

Disclaimer:

I will not talk directly about QCD...

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A. Czajka, S. Mrówczyński, *N=4 Super Yang-Mills Plasma*, 1203.1856:

> *... The N=4 super Yang-Mills plasma is finally concluded to be very similar the QCD plasma of gluons and light quarks. The differences mostly reflect different numbers of degrees of freedom in the two systems.*

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Outline

Key question

$\mathcal{N} = 4$ **plasma versus QCD plasma**

The AdS/CFT setup

Small excitations of a uniform static (strongly coupled) plasma

Nonlinear regime

Beyond hydrodynamics – towards a 4D description

Key question:

Understand the **real-time** properties of strongly coupled gauge theory plasma

Consider plasma in $\mathcal{N}=4$ Super-Yang-Mills theory
Use AdS/CFT methods...

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$\mathcal{N} = 4$ plasma versus QCD plasma

Similarities:

- Deconfined phase
- Strongly coupled
- **No supersymmetry!**

Differences:

- No running coupling $\longrightarrow$ Even at very high energy densities the coupling remains strong
- (Exactly) conformal equation of state $\longrightarrow$ Differences close to T_c, no bulk viscosity... (but not that different around $1.5 - 2.5 T_c$)
- No confinement/deconfinement phase transition $\longrightarrow$ Plasma fireball cools indefinitely

Some of these differences can be lifted in more complicated versions of the AdS/CFT correspondence

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How to describe strongly coupled plasma using AdS/CFT?

The AdS/CFT setup ($T_{\mu\nu}$ – geometry dictionary)

Method: Describe the (possibly time dependent) strongly coupled plasma system through a dual 5D geometry — given e.g. by

$$ds^2 = \frac{g_{\mu\nu}(x^\rho, z) dx^\mu dx^\nu + dz^2}{z^2} \equiv g^{5D}_{\alpha\beta} dx^\alpha dx^\beta$$

i) use Einstein's equations as field equations for $g_{\mu\nu}(x^\rho, z)$

$$R_{\alpha\beta} - \frac{1}{2} g^{5D}_{\alpha\beta} R - 6\, g^{5D}_{\alpha\beta} = 0$$

ii) read off $\langle T_{\mu\nu}(x^\rho) \rangle$ from the numerical metric $g_{\mu\nu}(x^\rho, z)$

$$g_{\mu\nu}(x^\rho, z) = \eta_{\mu\nu} + z^4 g^{(4)}_{\mu\nu}(x^\rho) + \ldots \qquad \langle T_{\mu\nu}(x^\rho) \rangle = \frac{N_c^2}{2\pi^2} \cdot g^{(4)}_{\mu\nu}(x^\rho)$$

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Example: Static uniform plasma

- Start from a constant diagonal energy momentum tensor (with $E = 3p$)

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix}$$

- Solve Einstein's equations with the above boundary condition for $g_{\mu\nu}(x^\rho, z)$...
- The result is a black hole geometry

$$ds^2 = -\frac{(1 - z^4/z_0^4)^2}{(1 + z^4/z_0^4)z^2}\, dt^2 + (1 + z^4/z_0^4)\frac{dx^2}{z^2} + \frac{dz^2}{z^2}$$

with z_0 expressed in terms of E $\left(E = \frac{3N_c^2}{2\pi^2 z_0^4}\right)$

- Hawking temperature $T_H = \frac{\sqrt{2}}{\pi z_0} \equiv$ gauge theory temperature

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What kind of excitations propagate in a static uniform strongly coupled plasma system?

Answer 1: Hydrodynamics

- The energy-momentum tensor $T_{\mu\nu}$ is expressed in terms of a local temperature T and flow velocity u^μ
- $T_{\mu\nu}$ is expressed as an expansion in the gradients of the flow velocities (shown here for $\mathcal{N} = 4$ SYM)

$$T^{\mu\nu}_{rescaled} = \underbrace{(\pi T)^4(\eta^{\mu\nu} + 4u^\mu u^\nu)}_{perfect\ \ fluid} - \underbrace{2(\pi T)^3\sigma^{\mu\nu}}_{viscosity} +$$

$$+ \underbrace{(\pi T^2)\left(\log 2\, T^{\mu\nu}_{2a} + 2\, T^{\mu\nu}_{2b} + (2 - \log 2)\left(\frac{1}{3} T^{\mu\nu}_{2c} + T^{\mu\nu}_{2d} + T^{\mu\nu}_{2e}\right)\right)}_{second\ \ order\ \ hydrodynamics}$$

- Consider small perturbations

$$T_{\mu\nu} = \begin{pmatrix} E & 0 & 0 & 0 \\ 0 & p & 0 & 0 \\ 0 & 0 & p & 0 \\ 0 & 0 & 0 & p \end{pmatrix} + \delta T_{\mu\nu} e^{-i\omega t + ikx}$$

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- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
  $$\omega_{shear} = -i \frac{\eta}{E+p} k^2$$
  sound modes:
  $$\omega_{sound} = \frac{1}{\sqrt{3}} k - i \frac{2}{3} \frac{\eta}{E+p} k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

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- *If* $T_{\mu\nu}$ is described by (1^{st} order viscous) hydrodynamics then one can derive dispersion relation of long wavelength modes from hydrodynamic equations:
  shear modes:
  $$\omega_{shear} = -i \frac{\eta}{E+p} k^2$$
  sound modes:
  $$\omega_{sound} = \frac{1}{\sqrt{3}} k - i \frac{2}{3} \frac{\eta}{E+p} k^2$$
- If we were to include terms in $T_{\mu\nu}$ with more derivatives (higher order viscous hydrodynamics), we would get terms with higher powers of k in the dispersion relations...
- Hypothetical resummed *all-order* hydrodynamics would predict the full dispersion relation for these modes $\omega_{shear}(k)$, $\omega_{sound}(k)$

What happens in the AdS/CFT description?

Answer 2: AdS/CFT

- Small disturbances of the uniform static plasma $\equiv$ small perturbations of the black hole metric ($\equiv$ quasinormal modes (QNM))

$$g^{5D}_{\alpha\beta} = g^{5D,black\ hole}_{\alpha\beta} + \delta g^{5D}_{\alpha\beta}(z) e^{-i\omega t + ikx}$$

- Dispersion relation fixed by linearized Einstein's equations. Results for the sound channel

from Kovtun,Starinets hep-th/0506184

- This is equivalent to summing contributions from *all-order* viscous hydrodynamics
- But, **in addition**, there is an infinite set of higher QNM — effective degrees of freedom not contained in the hydrodynamic description at all!

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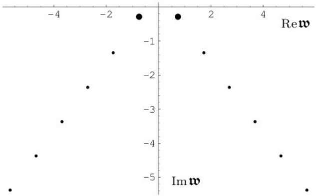

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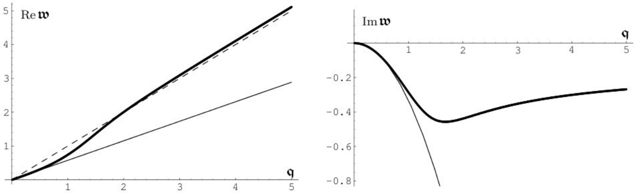

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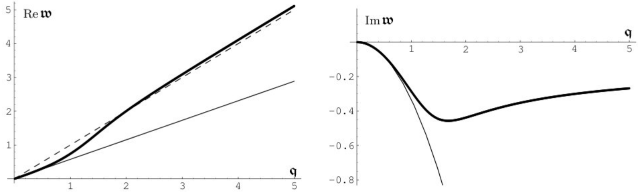

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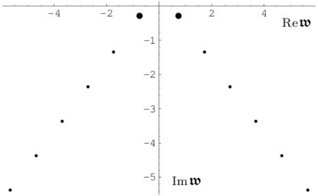

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- contain all-order viscous hydrodynamic modes (with specific values of all transport coefficients)

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- **in addition** contain the dynamics of genuine nonhydrodynamical modes

$$\omega^{(n)}_{non-hydro} = -i\Gamma_n \pm \Omega_n + \mathcal{O}\left(k^{\#}\right)$$

- incorporate their interactions in a fully nonlinear (and unique) way

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Einstein's equations can serve to study nonequilibrium processes in strongly coupled $\mathcal{N} = 4$ SYM and are an effective tool for exploring physics *beyond* hydrodynamics

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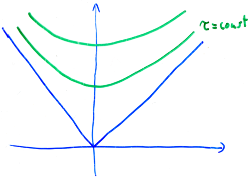
Bjorken '83

Assume a flow that is invariant under longitudinal boosts and does not depend on the transverse coordinates.



- In a conformal theory, $T^\mu_\mu = 0$ and $\partial_\mu T^{\mu\nu} = 0$ determine, under the above assumptions, the energy-momentum tensor completely in terms of a single function $\varepsilon(\tau)$, the energy density at mid-rapidity.
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$$p_L = -\varepsilon - \tau \frac{d}{d\tau}\varepsilon \quad \text{and} \quad p_T = \varepsilon + \frac{1}{2}\tau \frac{d}{d\tau}\varepsilon \,.$$

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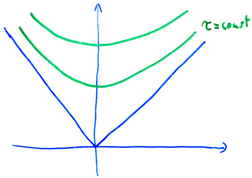

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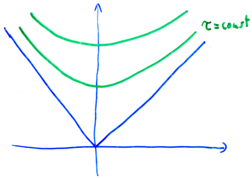

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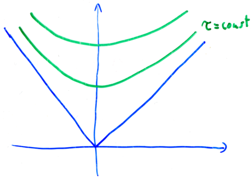

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Effective temperature T_{eff} as a function of τ

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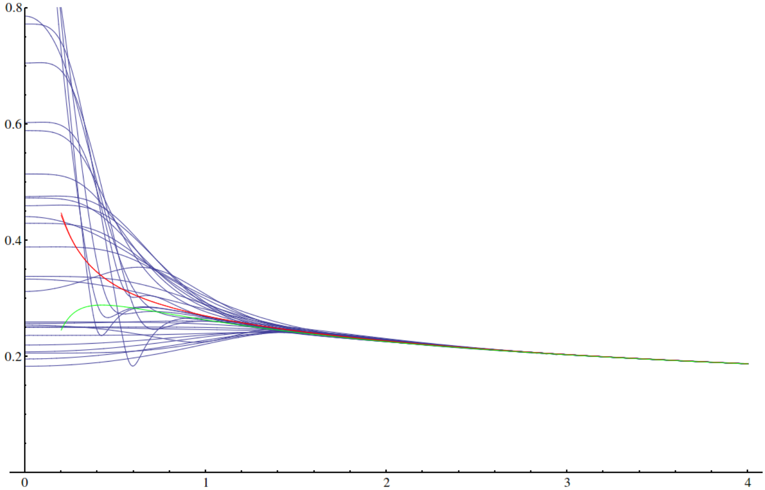

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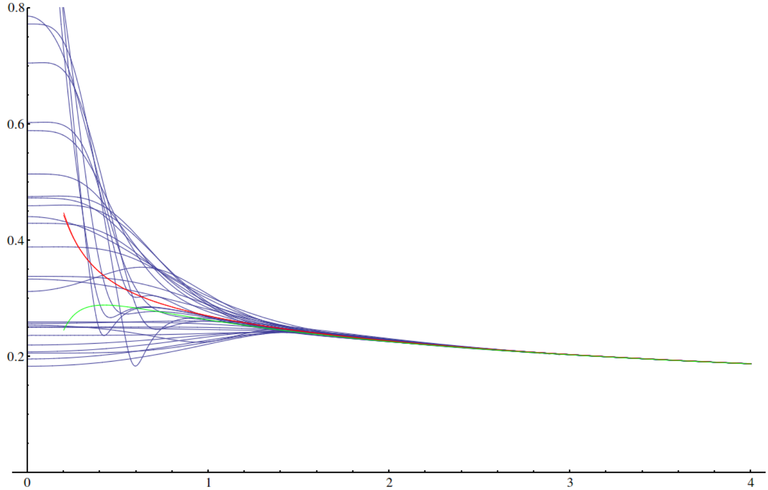

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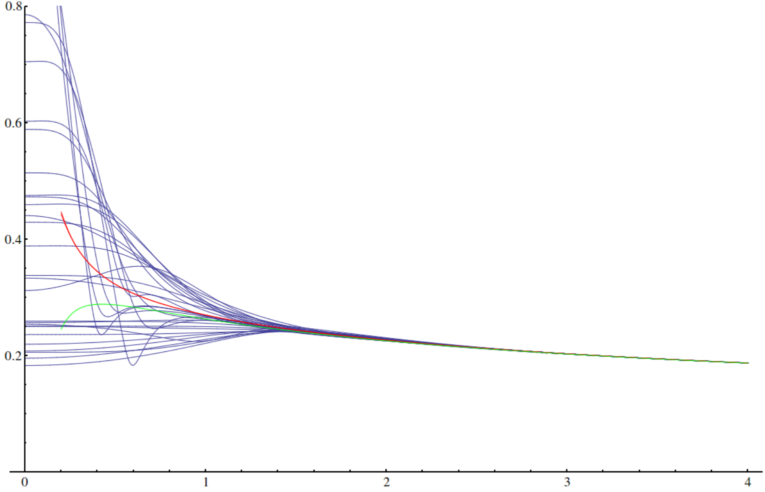

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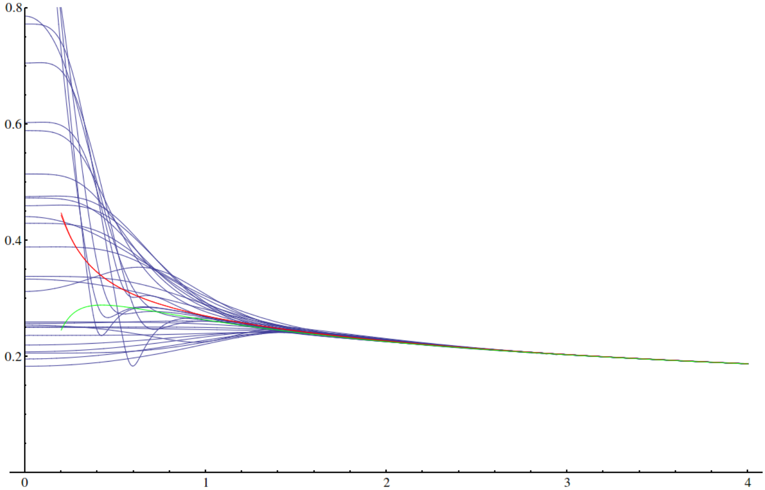

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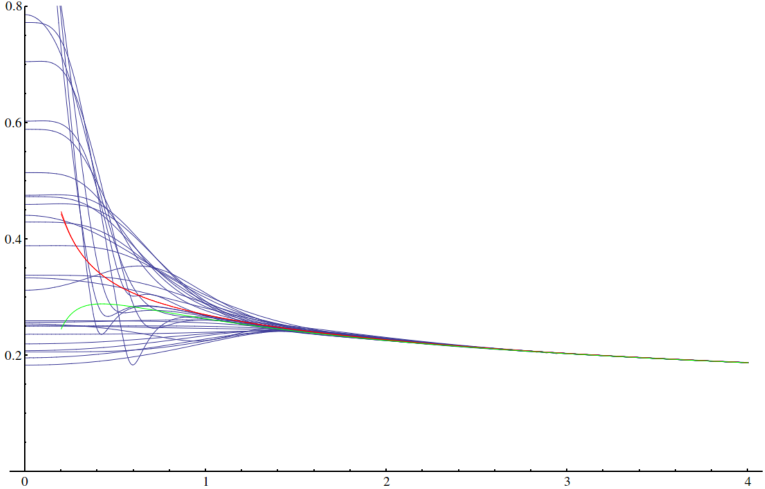

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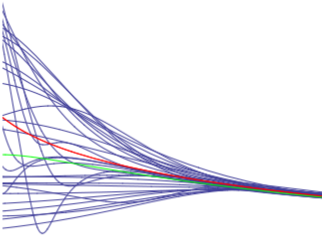

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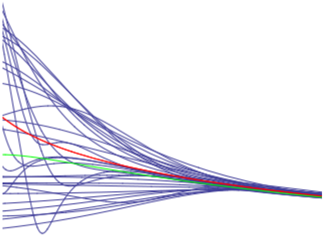

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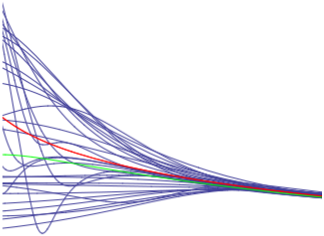

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- Consider a scalar degree of freedom ϕ (e.g. $\phi = \langle \mathrm{tr}\, F^2 \rangle$) in a generic hydrodynamic background
- Initially, let us consider the case of the background of static uniform plasma...
- On the AdS/CFT side, the scalar degree of freedom is described by quasinormal modes:

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- We will focus on the lowest (least damped) mode characterized by

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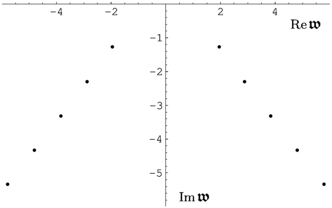

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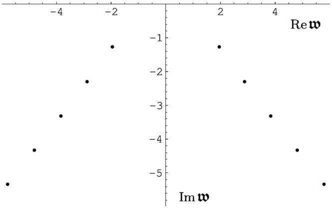

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$$\phi(t,x) = \int d^3k\, \alpha(k)\, e^{-\omega_I Tt} \cos(\omega_R Tt + \beta(k))\, e^{ikx}$$

- We can now deduce the 4D equation of motion obeyed by $\phi(t,x)$

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• **Key feature:** the frequencies have only very mild dependence on the spatial momentum — the dynamics seems **'ultralocal'**

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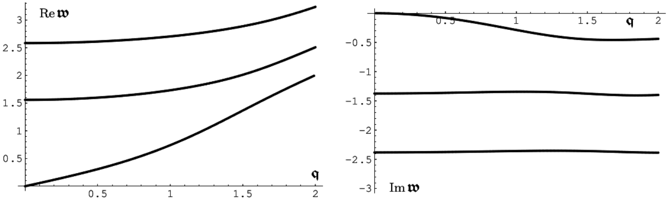

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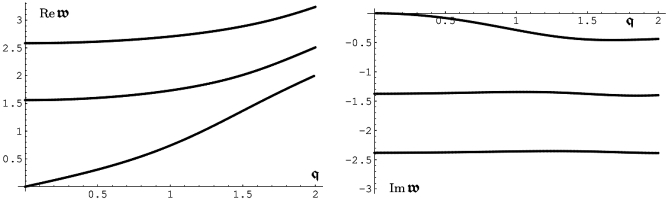

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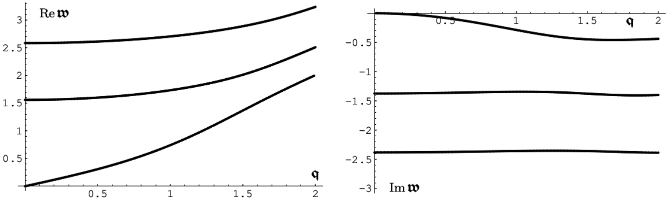

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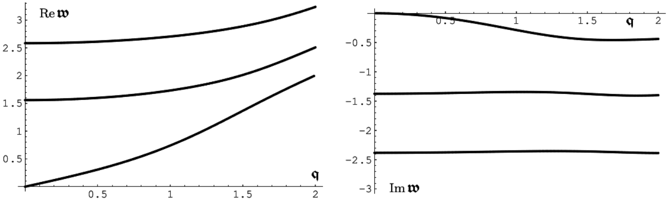

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- We aim at including the lowest nonhydrodynamic mode

static uniform plasma background

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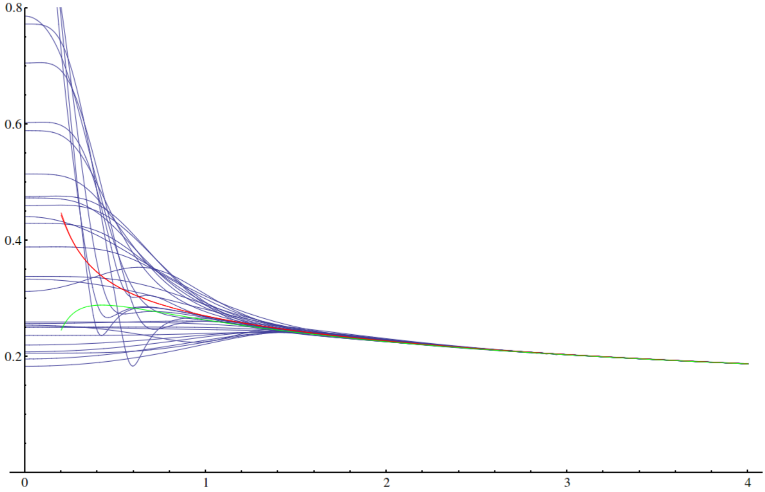
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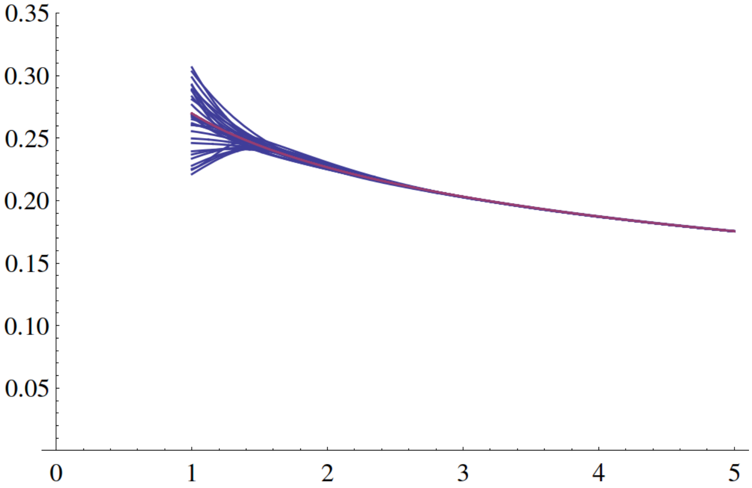
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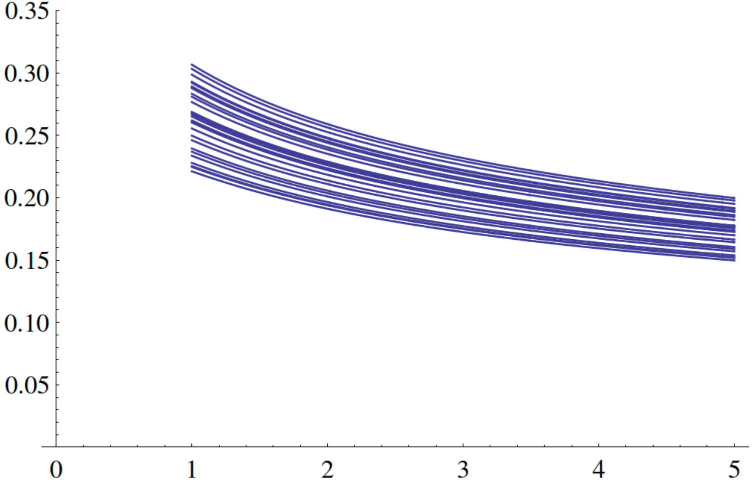
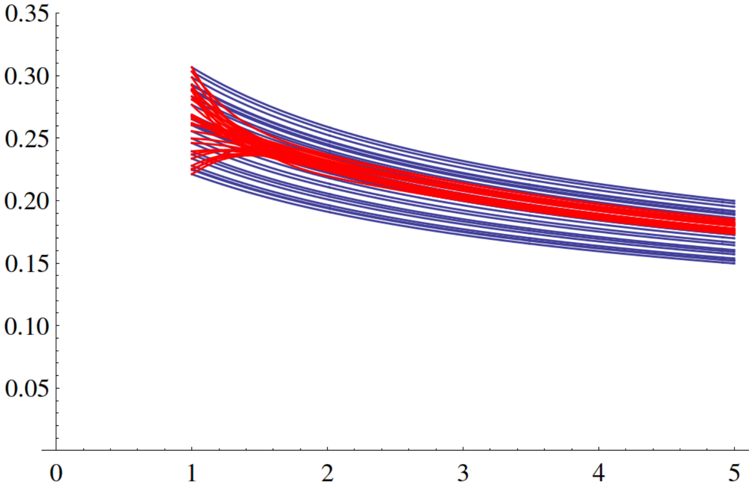


1^{st} order viscous hydrodynamic evolution



still preliminary...

Conclusions

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- Far from equilibrium these nonhydrodynamic degrees of freedom are very important for the dynamics
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