Renaissance of the sigma meson, is it interesting?

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UJK, Kielce, 10.05.2014

sigma meson: $f_0(500)$, former σ

- **What** is it? (basic)
- **Why** we are interested in?
- **How** we analyze it?
- **What** it really can be?

$f_0(500)$ *or* σ*:* **What** *is it?*

- scalar-isoscalar meson i.e. $J^{PC}I^G$: $0^{++}0^+$,
- lightest and widest: mass and width $\approx$ 500 MeV,
- hadronic decay channel: 100% $\pi\pi$,
- dramatic history:
  - until 1976 called ϵ or σ,
  - disappeared from Particle Data Tables between 1978 and 1992,
  - since 1994: $f_0(400 - 1200)$,
  - in years 2002-2010: $f_0(600)$,
  - now (since 2012): $f_0(500)$
- Renaissance of the sigma meson:

$$\boldsymbol{M_\sigma = Re(E_\sigma),\ \Gamma_\sigma = -2 \times Im(E_\sigma)}$$

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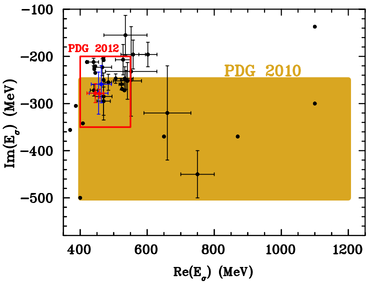
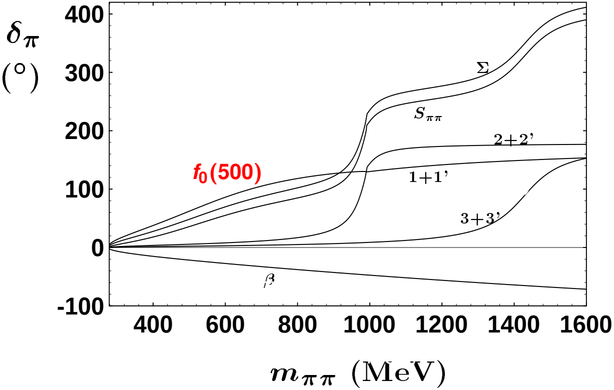


Why *we are interested in?*

- because it is a scalar-isoscalar meson and can be the lowest glueball state or mixture of $q\bar{q}$, $qq\bar{q}\bar{q}$ and gg,
- quite interesting neighborhood: $f_0(980) - K\bar{K}$ state?, $f_0(1370)$ - ?, $f_0(1500)$ - the lowest lattice gg meson,
- σ completely dominates the $\pi\pi$ threshold region,
- determines l_i constants needed in analyses of $q\bar{q}$ condensate,
- crucial for FSI in e.g. heavy meson decays $\longrightarrow$ CP violation, CKM matrix elements,
- difficult to study

Decomposition of the S0*-wave amplitude*

$$1 + 1' \longrightarrow f_0(500)$$
$$2 + 2' \longrightarrow f_0(980)$$
$$3 + 3' \longrightarrow f_0(1400)$$



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Measurement of $D_0 - \bar{D}_0$ *Mixing Parameters in* $D_0 \rightarrow K_s \pi^+ \pi^-$ *Decays*

PRL **99,** 131803 (2007) PHYSICAL REVIEW LETTERS week ending 28 SEPTEMBER 2007

TABLE I. Fit results and 95% C.L. intervals for x and y, including systematic uncertainties. The errors are statistical, experimental systematic, and decay-model systematic, respectively. For the *CPV*-allowed case, there is another solution as described in the text.

Fit case	Parameter	Fit result	95% C.L. interval
No	$x(\%)$	$0.80 \pm 0.29^{+0.09+0.10}_{-0.07-0.14}$	(0.0, 1.6)
CPV	$y(\%)$	$0.33 \pm 0.24^{+0.08+0.06}_{-0.12-0.08}$	(− 0.34, 0.96)
CPV	$x(\%)$	$0.81 \pm 0.30^{+0.10+0.09}_{-0.07-0.16}$	$\lvert x\rvert < 1.6$
	$y(\%)$	$0.37 \pm 0.25^{+0.07+0.07}_{-0.13-0.08}$	$\lvert y\rvert < 1.04$
	$\lvert q/p\rvert$	$0.86^{+0.30+0.06}_{-0.29-0.03} \pm 0.08$	...
	$\arg(q/p)(°)$	$-14^{+16+5+2}_{-18-3-4}$	...

events in the Q sideband $3\ \mathrm{MeV} < \lvert Q - 5.9\ \mathrm{MeV}\rvert < 14.1\ \mathrm{MeV}$.

For the combinatorial background, $\mathcal{P}_{\mathrm{cmb}}$ is the product of Dalitz plot and decay-time PDFs. The latter is parametrized as the sum of a delta function and an exponential function convolved with a Gaussian resolution function. The timing and Dalitz PDF parameters are obtained from fitting events in the mass sideband $30\ \mathrm{MeV}/c^2 < \lvert m_{K_S^0\pi\pi} - m_{D^0}\rvert < 55\ \mathrm{MeV}/c^2$.

The likelihood function for $\bar{D}^0$ decays, $\bar{\mathcal{L}}$, has the same form as $\mathcal{L}$, with $\mathcal{M}$ and $\bar{\mathcal{M}}$ (appearing in $\mathcal{P}_{\mathrm{sig}}$ and $\mathcal{P}_{\mathrm{rnd}}$) interchanged. To determine x and y, we maximize the sum $\ln\mathcal{L} + \ln\bar{\mathcal{L}}$. Table I lists the results from two separate fits.

TABLE II. Fit results for Dalitz-plot parameters. The errors are statistical only.

Resonance	Amplitude	Phase (deg)	Fit fraction
$K^*(892)^-$	1.629 ± 0.006	134.3 ± 0.3	0.6227
$K_0^*(1430)^-$	2.12 ± 0.02	-0.9 ± 0.8	0.0724
$K_2^*(1430)^-$	0.87 ± 0.02	-47.3 ± 1.2	0.0133
$K^*(1410)^-$	0.65 ± 0.03	111 ± 4	0.0048
$K^*(1680)^-$	0.60 ± 0.25	147 ± 29	0.0002
$K^*(892)^+$	0.152 ± 0.003	-37.5 ± 1.3	0.0054
$K_0^*(1430)^+$	0.541 ± 0.019	91.8 ± 2.1	0.0047
$K_2^*(1430)^+$	0.276 ± 0.013	-106 ± 3	0.0013
$K^*(1410)^+$	0.33 ± 0.02	-102 ± 4	0.0013
$K^*(1680)^+$	0.73 ± 0.16	103 ± 11	0.0004
$\rho(770)$	1 (fixed)	0 (fixed)	0.2111
$\omega(782)$	0.0380 ± 0.0007	115.1 ± 1.1	0.0063
$f_0(980)$	0.380 ± 0.004	-147.1 ± 1.1	0.0452
$f_0(1370)$	1.46 ± 0.05	98.6 ± 1.8	0.0162
$f_2(1270)$	1.43 ± 0.02	-13.6 ± 1.2	0.0180
$\rho(1450)$	0.72 ± 0.04	41 ± 7	0.0024
σ_1	1.39 ± 0.02	-146.6 ± 0.9	0.0914
σ_2	0.267 ± 0.013	-157 ± 3	0.0088
NR	2.36 ± 0.07	155 ± 2	0.0615

solution. From the fit to data, we find that the Dalitz plot parameters are consistent for the D^0 and $\bar{D}^0$ samples; hence we observe no evidence for direct *CPV*. Results for $\lvert p/q\rvert$ and $\arg(p/q)$, parameterizing *CPV* in mixing and interference between mixed and unmixed amplitudes, respectively, are also found to be consistent with *CP*

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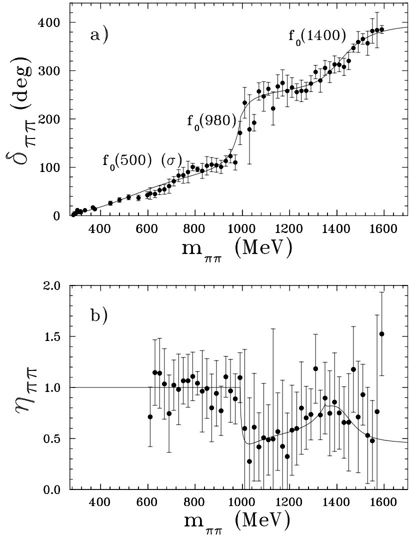
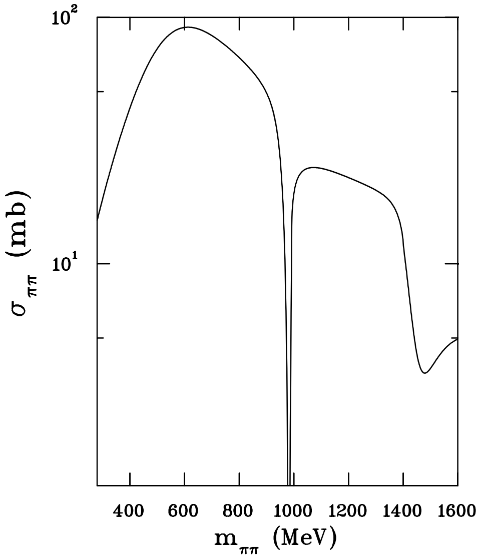
$\pi\pi$ S0-wave phase shifts and inelasticities



Puzzling S0 wave ππ cross section



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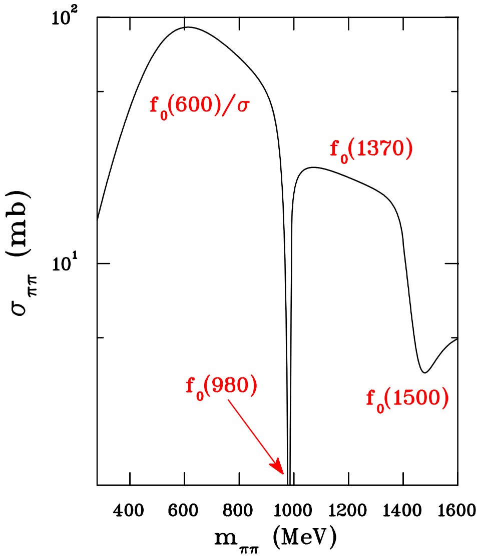
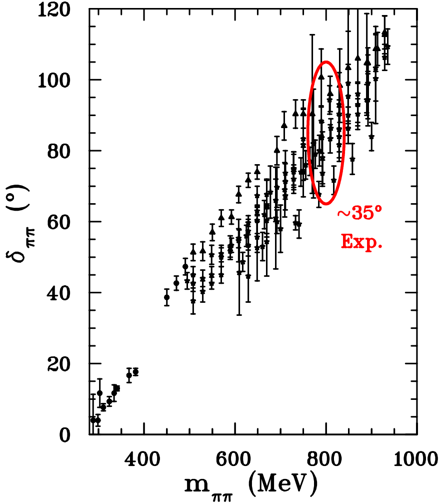


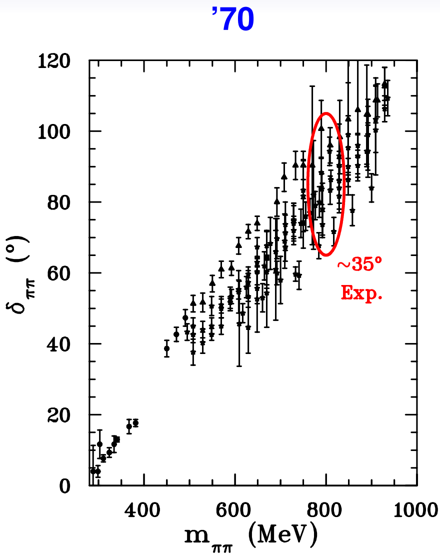
GKPY dispersion equations with imposed

’70



GKPY dispersion equations with imposed crossing symmetry condition

Madrid-Kraków group 2005-2011



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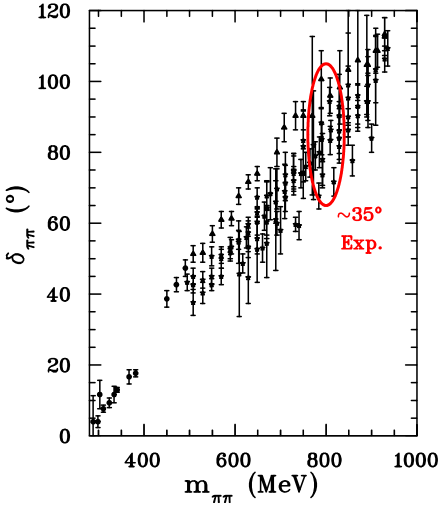

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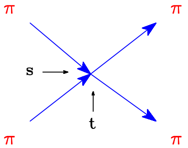
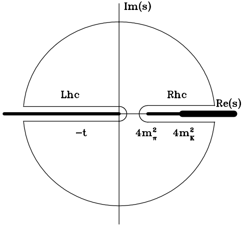
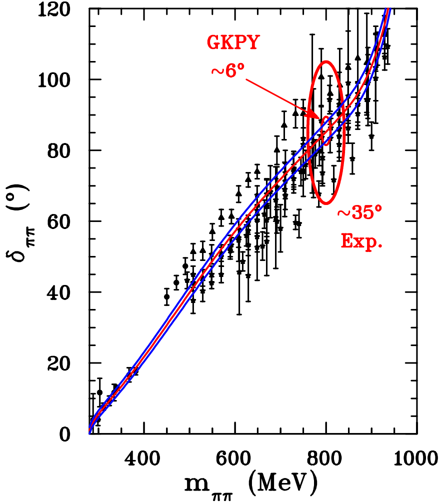




'70 ⟶ 2011



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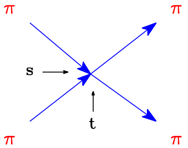
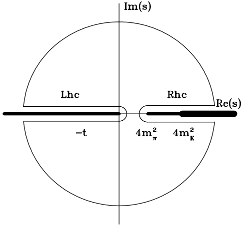




GKPY equations and poles of the $\pi\pi$ amplitudes

partial waves: *JI*

experiment

F1 D2

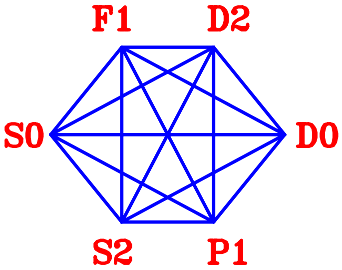
S0 D0

S2 P1

GKPY equations and poles of the $\pi\pi$ amplitudes

partial waves: *JI*

experiment + theory (GKPY)

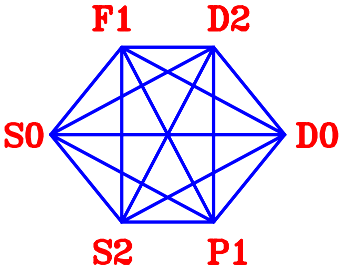


GKPY equations and poles of the $\pi\pi$ amplitudes

partial waves: *JI*

experiment + theory (GKPY)

$$\int_{4m_\pi^2}^{\infty}$$



GKPY equations:

$$\text{Re}\, t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C^{II'}\, t_0^{I'(IN)}(4m_\pi^2) + \sum_{I'=0}^{2} \sum_{\ell'=0}^{4} \int_{4m_\pi^2}^{\infty} ds'\, K_{\ell\ell'}^{II'}(s,s')\, \text{Im}\, t_{\ell'}^{I'(IN)}(s')$$

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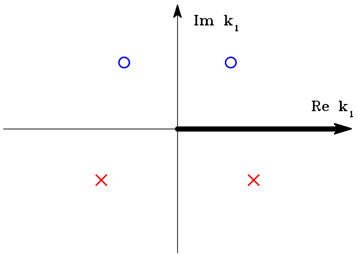
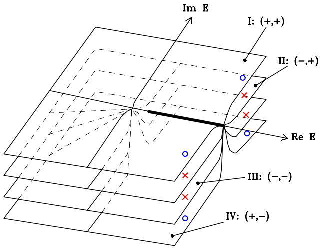
and poles of the $\pi\pi$ amplitudes:

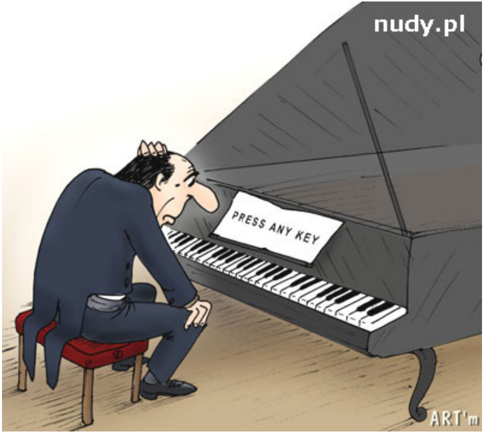




We had to be: well equipped ...

We had to check everything ....

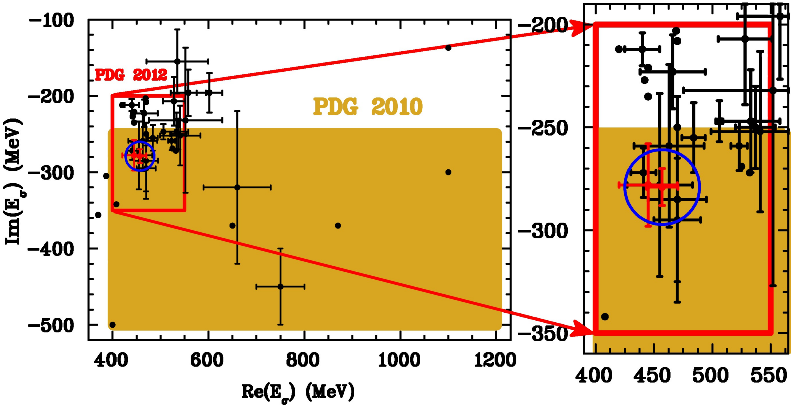
.. and sometimes we were without any idea ....



sometimes we were misled ....

anyway we had to work very hard and finally were very tired ....

$$M = Re(E_{pole}), \quad \Gamma = -2 \times Im(E_{pole})$$



Before 2012

Citation: C. Amsler *et al.* (Particle Data Group), PL **B667**, 1 (2008) and 2009 partial update for the 2010 edition (URL: http://pdg.lbl.gov)

$f_0(600)$ or σ $I^G(J^{PC}) = 0^+($

A REVIEW GOES HERE – Check our WWW

$f_0(600)$ T-MATRIX POLE $\sqrt{s}$

Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*		*TECN*
(400–1200)$-i$(250–500) OUR ESTIMATE			
• • • We do not use the following data for averages, fits, limits, et			
$(455 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$	[1] CAPRINI	08	RVUE
$(463 \pm 6^{+31}_{-17}) - i(259 \pm 6^{+33}_{-34})$	[2] CAPRINI	08	RVUE
$(552^{+84}_{-106}) - i(232^{+81}_{-72})$	[3] ABLIKIM	07A	BES2
$(466 \pm 18) - i(223 \pm 28)$	[4] BONVICINI	07	CLEO
$(484 \pm 17) - i(255 \pm 10)$	GARCIA-MAR..	07	RVUE
$(441^{+16}_{-8}) - i(272^{+9}_{-12.5})$	[5] CAPRINI	06	RVUE
$(470 \pm 50) - i(285 \pm 25)$	[6] ZHOU	05	RVUE
$(541 \pm 39) - i(252 \pm 42)$	[7] ABLIKIM	04A	BES2
$(528 \pm 32) - i(207 \pm 23)$	[8] GALLEGOS	04	RVUE
$(440 \pm 8) - i(212 \pm 15)$	[9] PELAEZ	04A	RVUE
$(533 \pm 25) - i(247 \pm 25)$	[10] BUGG	03	RVUE
$532 - i272$	BLACK	01	RVUE
$(470 \pm 30) - i(295 \pm 20)$	[5] COLANGELO	01	RVUE

Since year 2012

Citation: J. Beringer *et al.* (Particle Data Group), PR **D86**, 010001 (2012) and 2013 p

$f_0(500)$ or σ was $f_0(600)$ $I^G(J^{PC}$

A REVIEW GOES HERE – Check our

$f_0(500)$ T-MATRIX POL

Note that $\Gamma \approx 2\ \mathrm{Im}(\sqrt{s_{\text{pole}}})$.

VALUE (MeV)	*DOCUMENT ID*	
(400–550)$-i$(200–350) OUR ESTIMATE		
• • • We do not use the following data for averages, fits		
$(440 \pm 10) - i(238 \pm 10)$	[1] ALBALADEJO	12
$(445 \pm 25) - i(278^{+22}_{-18})$	[2,3] GARCIA-MAR..	11
$(457^{+14}_{-13}) - i(279^{+11}_{-7})$	[2,4] GARCIA-MAR..	11
$(442^{+5}_{-8}) - i(274^{+6}_{-5})$	[5] MOUSSALLAM	11
$(452 \pm 13) - i(259 \pm 16)$	[6] MENNESSIER	10
$(448 \pm 43) - i(266 \pm 43)$	[7] MENNESSIER	10
$(455 \pm 6^{+31}_{-13}) - i(278 \pm 6^{+34}_{-43})$	[8] CAPRINI	08
$(463 \pm 6^{+31}_{-17}) - i(259 \pm 6^{+33}_{-34})$	[9] CAPRINI	08
$(552^{+84}_{-106}) - i(232^{+81}_{-72})$	[10] ABLIKIM	07
$(466 \pm 18) - i(223 \pm 28)$	[11] BONVICINI	07
$(472 \pm 30) - i(271 \pm 30)$	[12] BUGG	07
$(484 \pm 17) - i(255 \pm 10)$	GARCIA-MAR..	07

Roy's equations and up-down ambiguity in the $\pi\pi$ S0 wave

$$\text{Re}\, t_\ell^{I(OUT)}(s) = a_0^0 + (2a_0^0 - 5a_0^2)(s-4) + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \int_{4m_\pi^2}^{\infty} ds'\, \bar{K}_{\ell\ell'}^{II'}(s,s')\, \text{Im}\, t_{\ell'}^{I'(IN)}(s')$$

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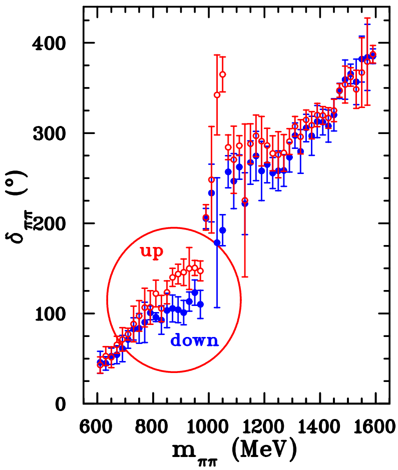

precision of the Roy and GKPY equations

Roy' 1971	GKPY' 2011
two subtractions	one subtraction
$K_{\ell\ell'}^{ll'}(s,s') \sim s'^{-3}$-fast convergence	$K_{\ell\ell'}^{ll'}(s,s') \sim s'^{-2}$
$ST_0^0 = a_0^0 + (2a_0^0 - 5a_0^2)(s-4)$	$ST_0^0 = a_0^0 + 5a_0^2$ - no error propagation!

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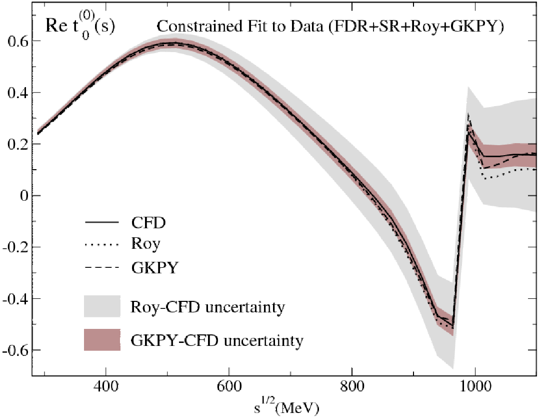


... for sure your solution is not unique

Another group - "Bern" group:
H. Leytwyller, J. Gasser, G. Colangelo, I. Caprini ...

The Role of the input in Roy's equations for pi pi scattering" G. Wanders, Eur. Phys. J. C17 (2000) 323-336

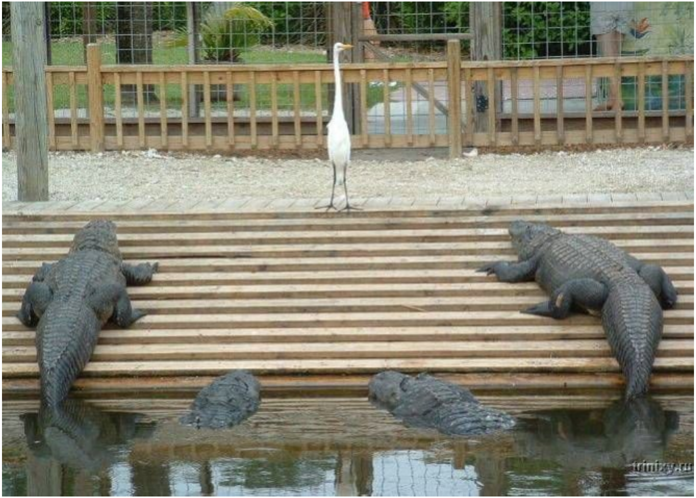
In the abstract:

An updated survey of known results on the dimension of the manifold of solutions is presented. The solution is unique for a low energy interval with upper end at 800 MeV. We determine its response to small variations of the input: S-wave scattering lengths and absorptive parts above 800 MeV.

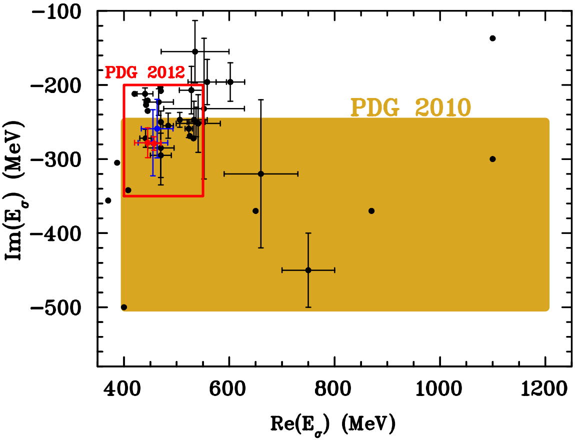
I.e.:
Fixed two boundary conditions for the $\pi\pi$ amplitude:

- at the threshold ($S0$ wave scattering length) and
- at 800 MeV

tiny error bands: common target



Bern and Madrid groups finally agreed ...



specific choice of the parameterization?

Madrid: $cot\delta_0^0 = \frac{\sqrt{s}}{2k}\frac{M_\pi^2}{s-\frac{1}{2}z_0^2}\left[B_0 + B_1 w(s) + B_2 w(s)^2 + B_3 w(s)^3\right], \quad w = \frac{\sqrt{s}-\sqrt{s_0-s}}{\sqrt{s}+\sqrt{s_0-s}}$

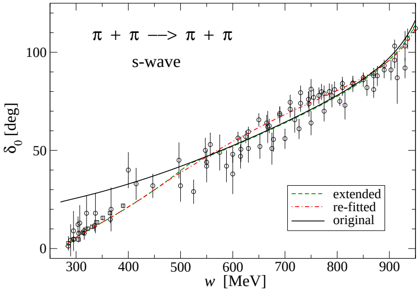
Test amplitude: $T(s) \sim \prod_{i=1}^{N}\left[w(s) - w_i\right], \quad w = \frac{\sqrt{s-s_2}+\sqrt{s-s3}}{\sqrt{s_3-s_2}}$

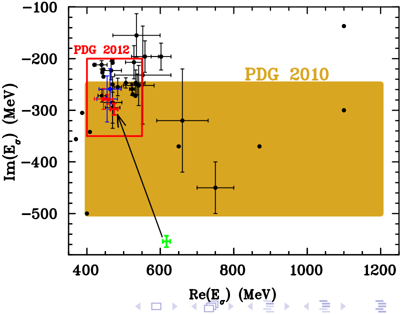
New low energy amplitude (up to $\sim 400 - 500$ MeV):

$$Ref_\ell^I(s) = \frac{\sqrt{s}}{4k} sin2\delta_\ell^I = m_\pi k^{2l}[a_\ell^I + b_\ell^I k^2 + c_\ell^I k^4 + d_\ell^I k^6 + O(k^8)]$$

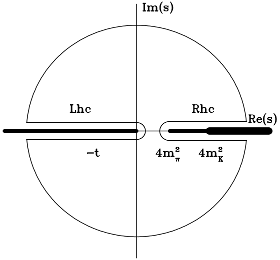
above $\sim 400 - 500$ MeV - structure of amplitude not changed

repeated fit to the data (not changed) + GKPY equations





... left cut is enough, we do not need GKPY ...



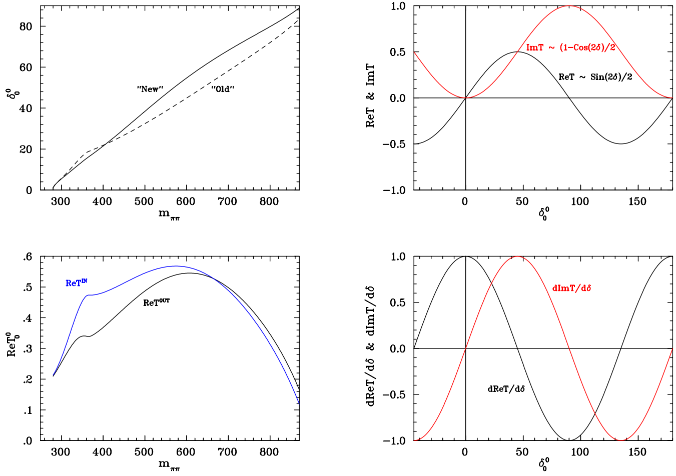
Left hand cut in parameterizations of amplitudes:

- additional factor $e^{i\alpha}$ in the full $S = e^{2i\delta}$ matrix element,
- It has, however, nothing to do with crossing symmetry!
    - It does not provide any type of relationship $A(s,t) = C_{st}A(t,s)$,
    - Moreover, subtracting constant is not specified so the output amplitude can be arbitrarily scaled!
- it makes amplitude only more realistic

what forces GKPY eqs to pull up-left the sigma pole?

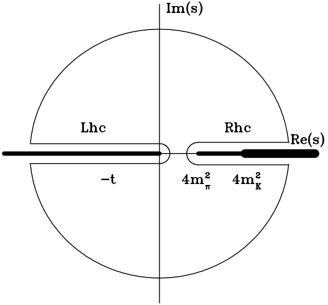
$$\mathrm{Re}\ t_\ell^{I(OUT)}(s) = \sum_{I'=0}^{2} C^{II'}\, t_0^{I'(IN)}(4m_\pi^2) + \sum_{I'=0}^{2}\sum_{\ell'=0}^{4} \int_{4m_\pi^2}^{\infty} ds'\, K_{\ell\ell'}^{II'}(s,s')\ \mathrm{Im}\ t_{\ell'}^{I'(IN)}(s')$$

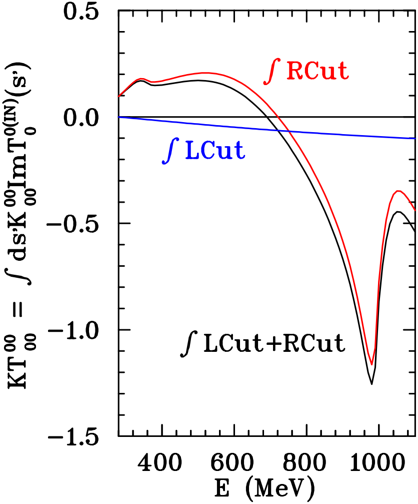
$$\mathrm{Re}\ t_0^{0(OUT)}(s) = \mathrm{Re}\ t_0^{0(IN)}(s)$$



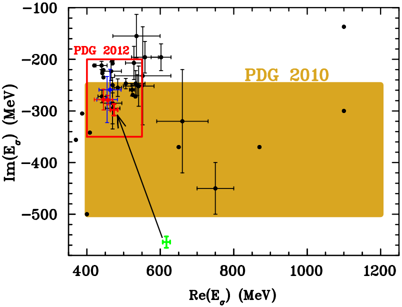
What does lead to such shape of the KT^{00}_{00}?

The shape is given by coefficients in the crossing symmetry matrix C_{st} and integrated amplitudes. Is it produced by the integration along the left or right cut?





what forces GKPY eqs to pull up-left the sigma pole?



Two things: trigonometry and crossing symmetry algebra lead to narrower and lighter σ.

Nothing more and nothing instead of it is needed.

What it really can be?

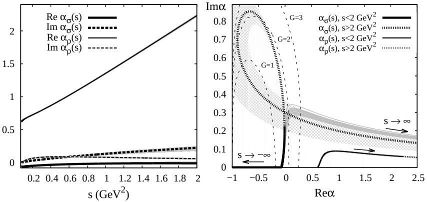


Fig. 1. (Left) $\alpha_\rho(s)$ and $\alpha_\sigma(s)$ Regge trajectories, from our constrained Regge-pole amplitudes. (Right) $\alpha_\sigma(s)$ and $\alpha_\rho(s)$ in the complex plane. At low and intermediate energies (thick continuous lines), the trajectory of the σ is similar to those of Yukawa potentials $V(r) = -Ga\exp(-r/a)/r$ [8] (thin dashed lines). Beyond 2 Gev^2 we plot our results as thick discontinuous lines because they should be considered just as extrapolations.

Furthermore, in Fig. 1 we show the striking similarities between the $f_0(500)$ trajectory and those of Yukawa potentials in non-relativistic scattering [8]. From the Yukawa G=2 curve in that plot, which lies closest to our result for the $f_0(500)$, we can estimate $a \simeq 0.5\,\text{GeV}^{-1}$, following [8]. This could be compared, for instance, to the S-wave $\pi\pi$ scattering length $\simeq 1.6\,\text{GeV}^{-1}$. Thus it seems that the range of a Yukawa potential that would mimic our low energy results is comparable but smaller than the $\pi\pi$ scattering length in the scalar isoscalar channel. Of course, our results are most reliable at low energies (thick continuous line) and the extrapolation should be interpreted cautiously. Nevertheless, our results suggest that the $f_0(500)$ looks more like a low-energy resonance of a short range potential, *e.g.* between pions, than a bound state of a confining force between a quark and an antiquark. ←

In summary, our formalism and the results for the $f_0(500)$ explains why the lightest scalar meson has to be excluded from the ordinary linear Regge fits of ordinary mesos.

"The non-ordinary Regge behavior of the $f_0(500)$ meson"

by

J. R. Pelaez, J. T. Londergan,
J . Nebreda and A. Szczepaniak

arXiv:1404.6058

Conclusions

- the σ meson is once again alive and is doing well!
- for sure σ is not pure $q\bar{q}$ meson but perhaps:
  - mixture of the $q\bar{q}$, $qq\bar{q}\bar{q}$ and gg components,
  - something like "correlated two-pion" state?
- opens a promising area for new analyses, especially for the $f_0(980)$, $f_0(1500)$ ...,
- should help end the debate about the existence of the $f_0(1370)$,
- it should help in precise determination of the CKM matrix elements and in the fight against the isobar model and old habits related with resonances