Uniwersytet Wrocławski

QCD in dense matter - prospects for and beyond NJL-kind effective models

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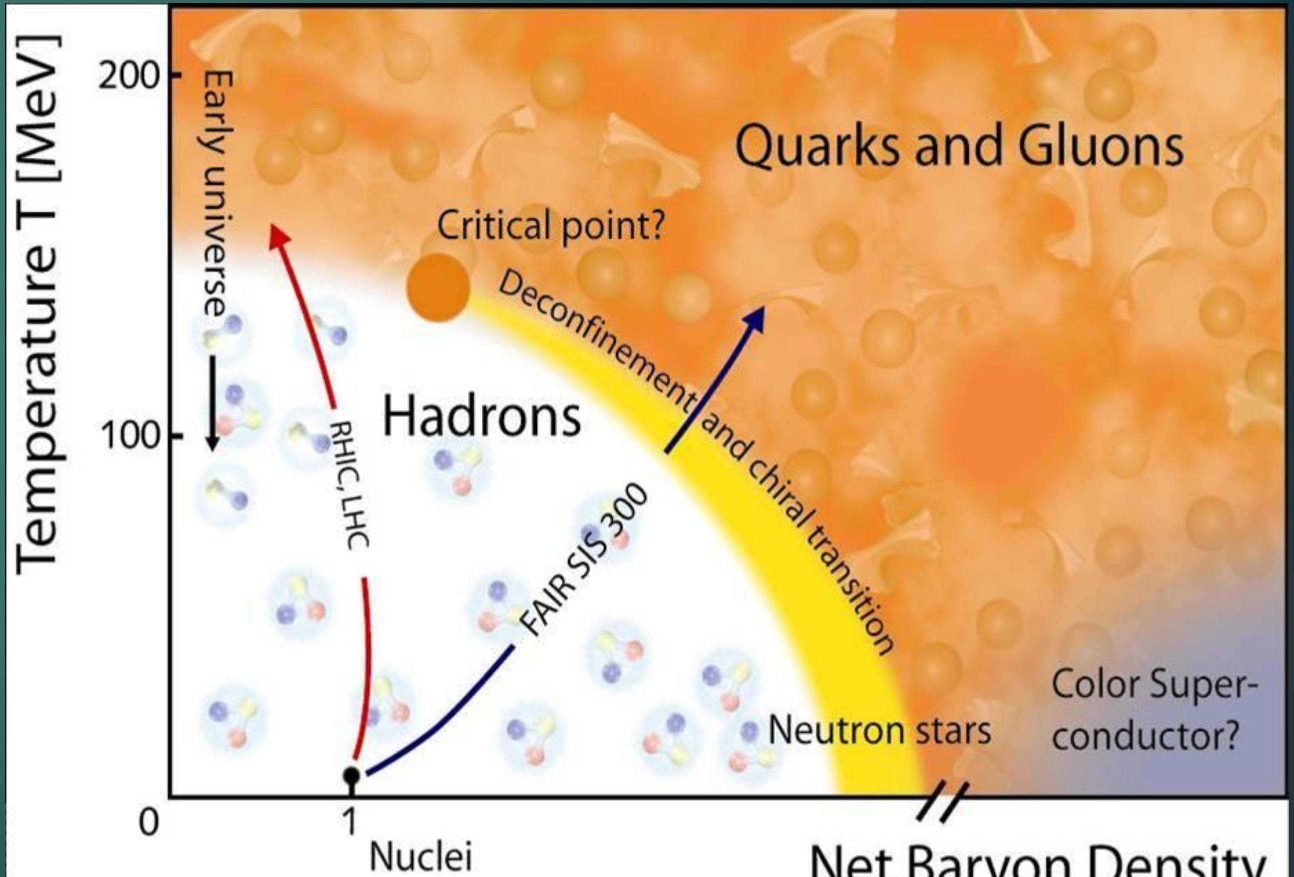
QCD Phase Diagram

- **dense baryonic matter**

HIC in collider experiments
Won't cover the whole diagram
Hot and 'rather' symmetric

NS as a 2nd accessible option
Cold and 'rather' asymmetric

Problem is more complex than
It looks at first gaze



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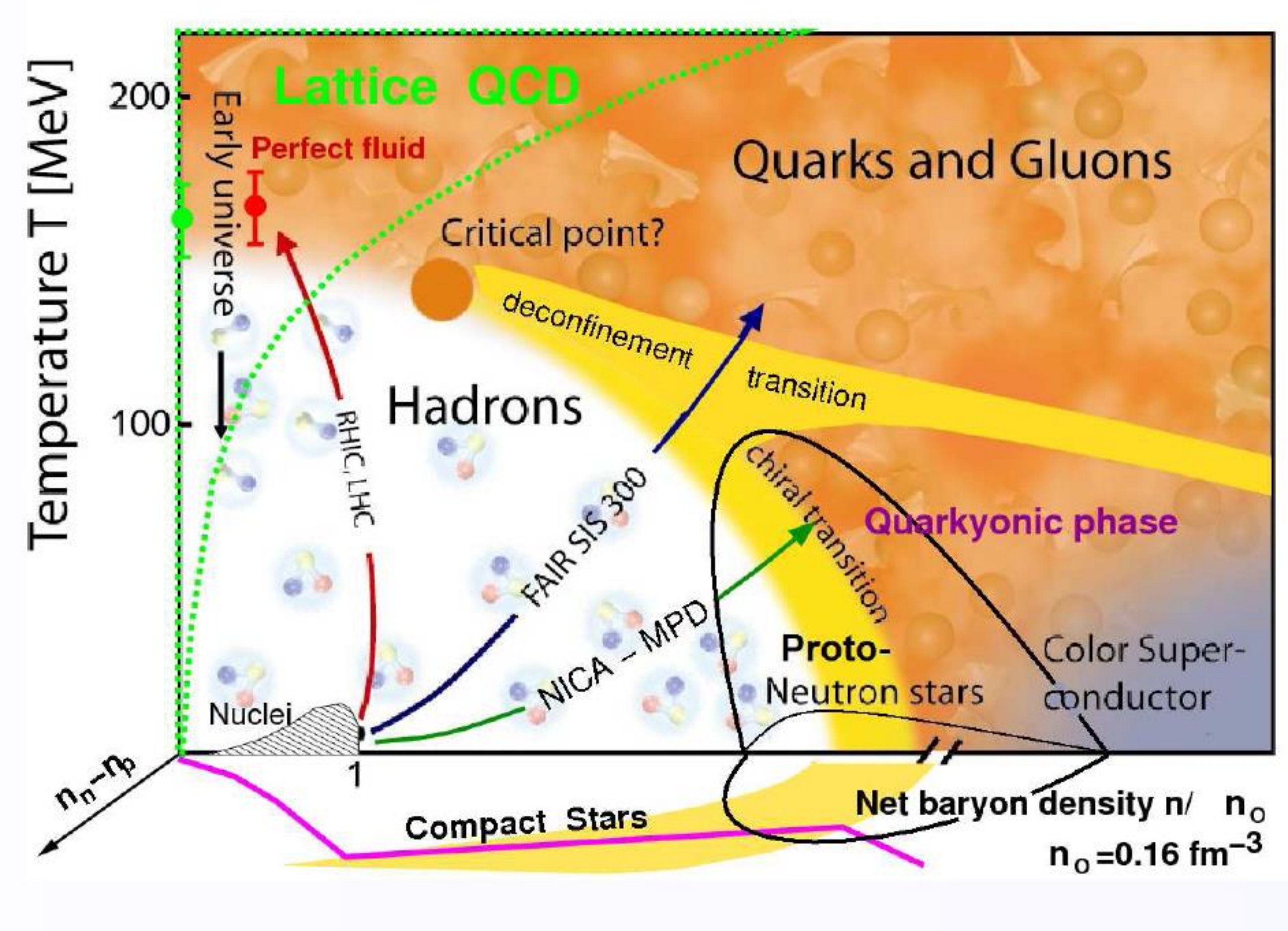
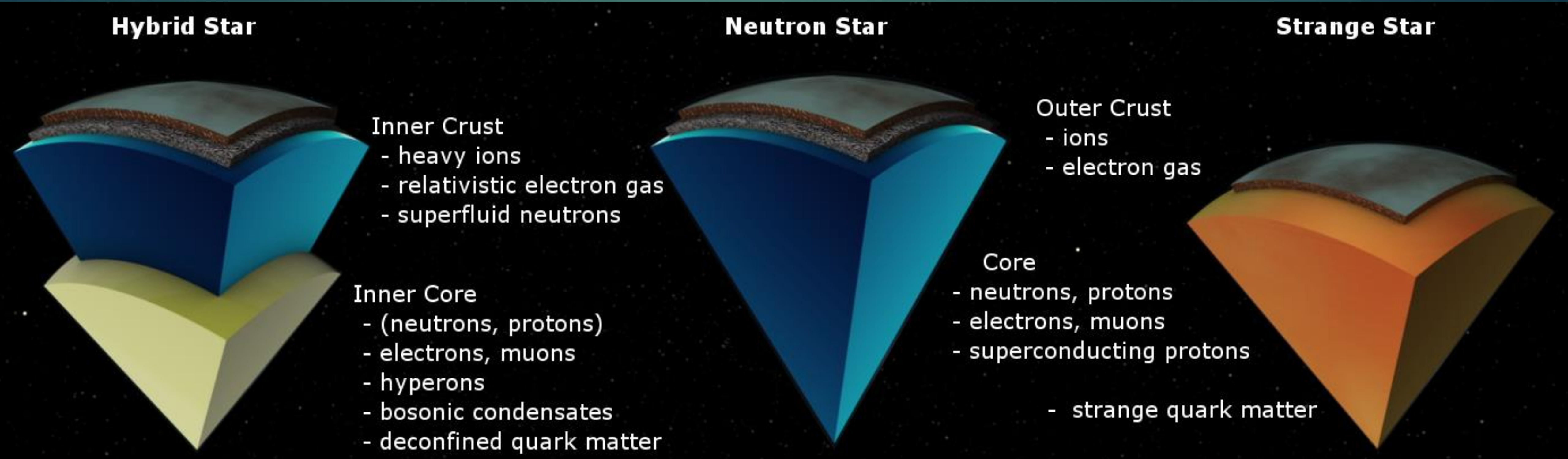
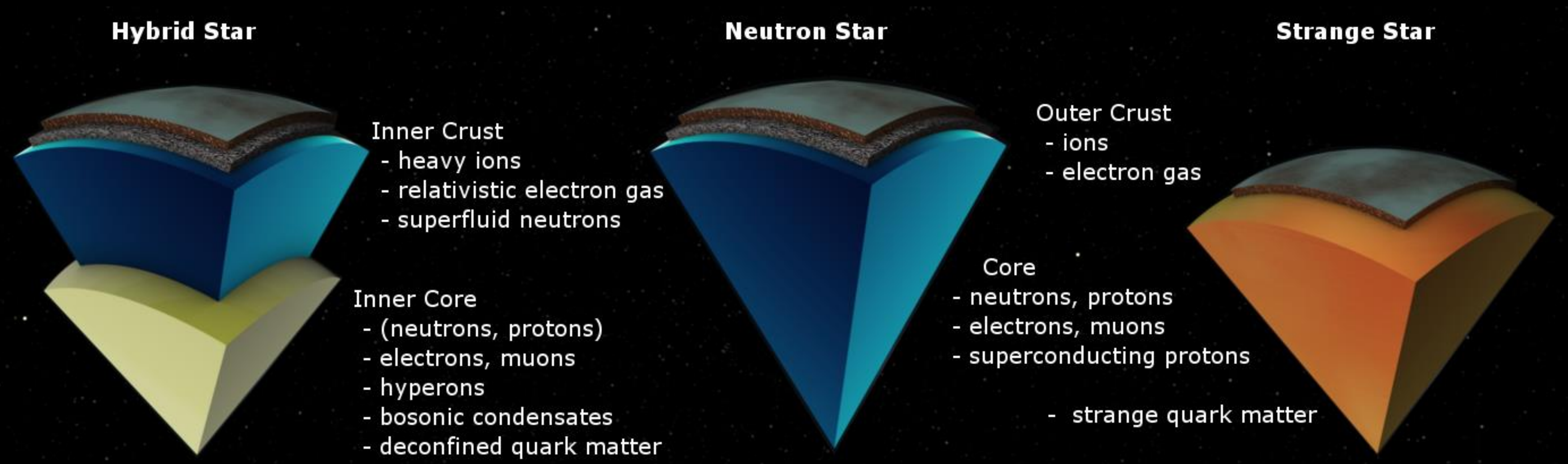


Neutron Stars

- Variety of scenarios regarding inner structure: with or without QM
- Question whether/how QCD phase transition occurs is not settled
- Most honest approach: take all possible scenarios into account and compare to available data



Neutron Stars = Quark Cores?



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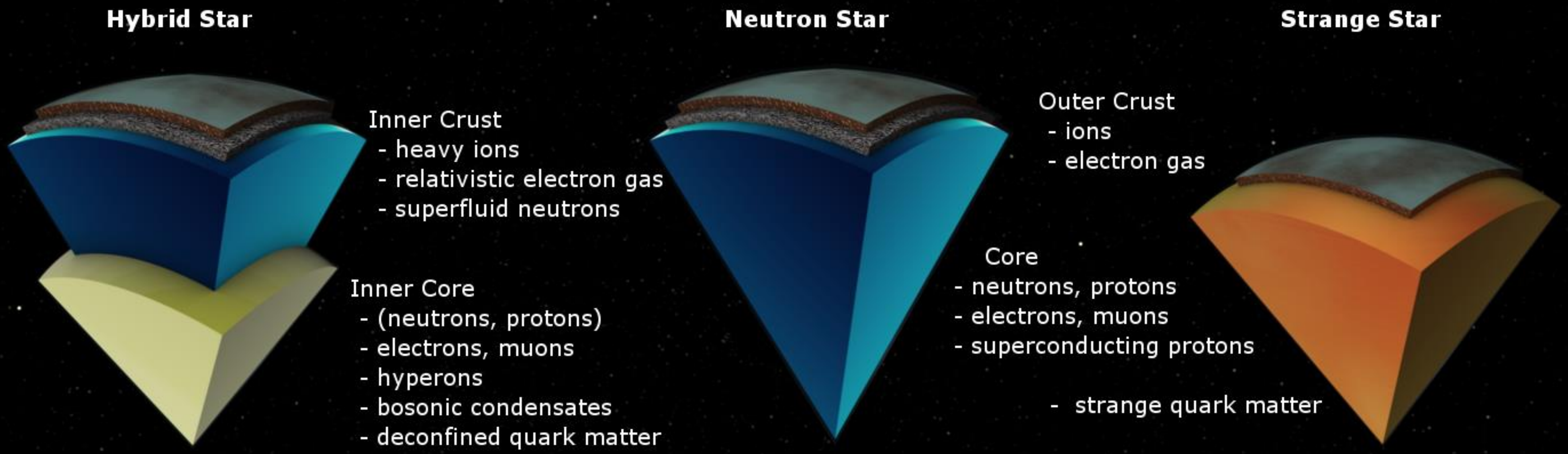

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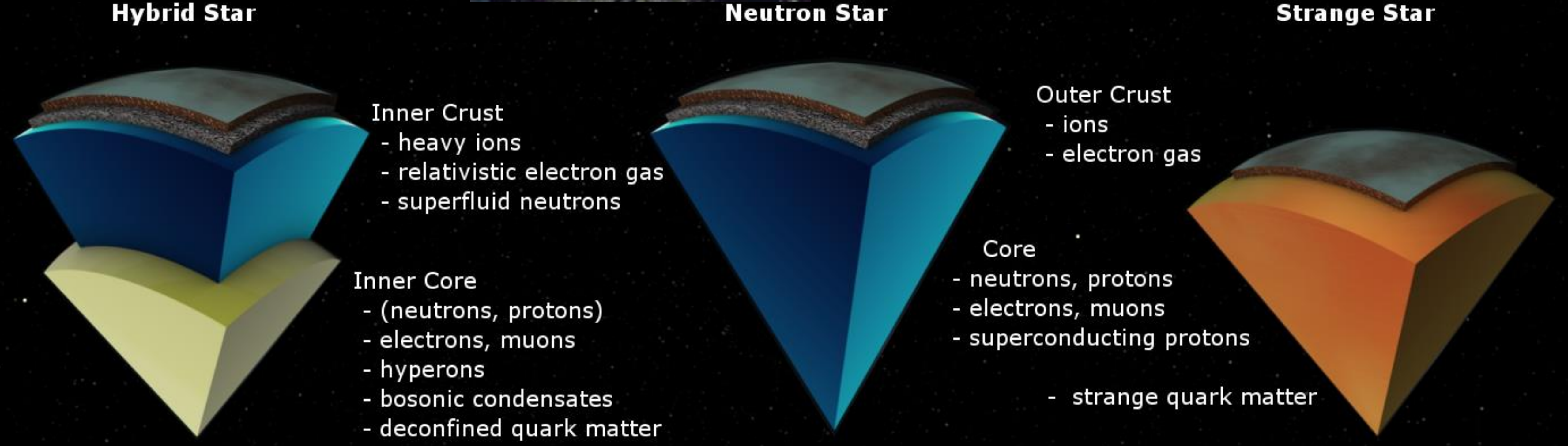

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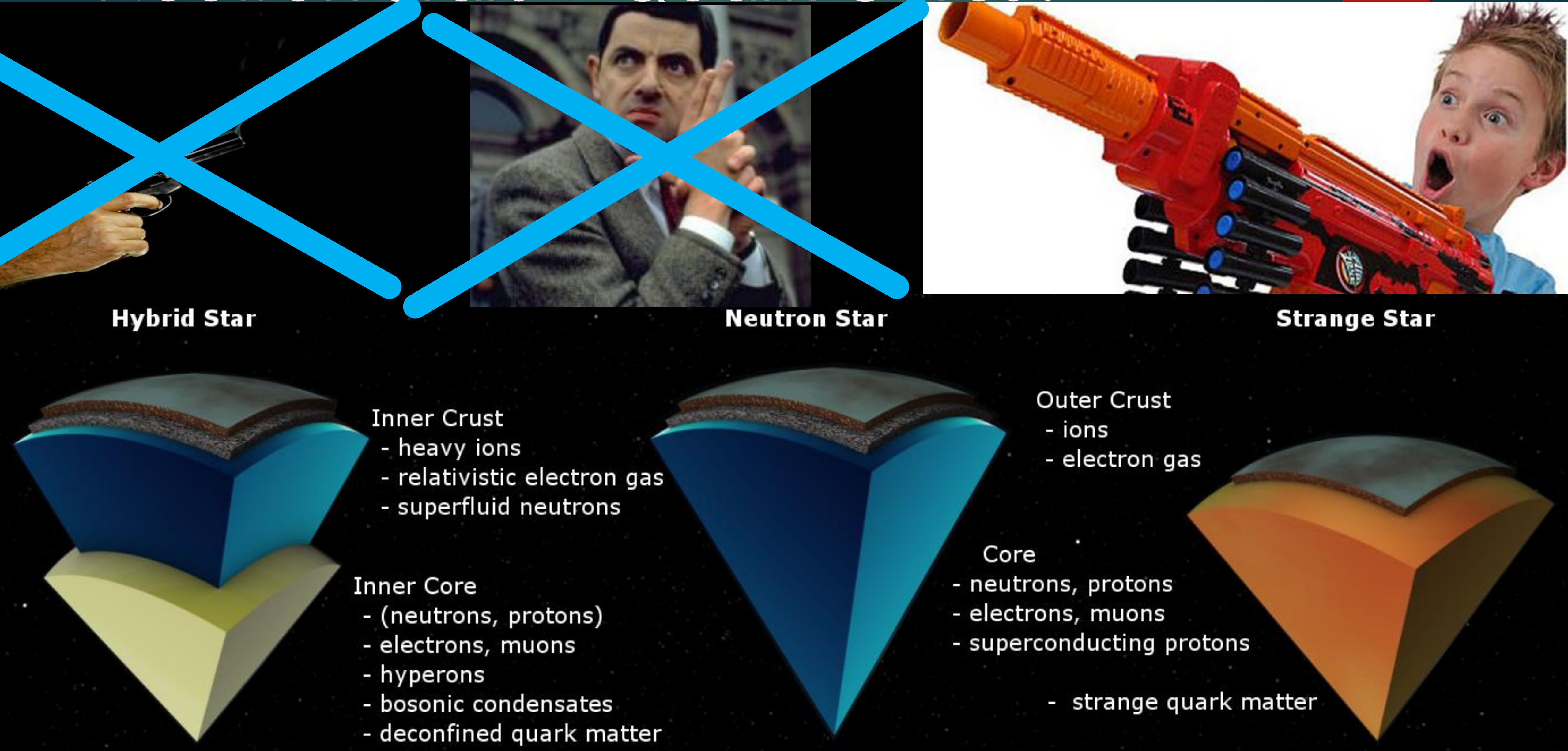
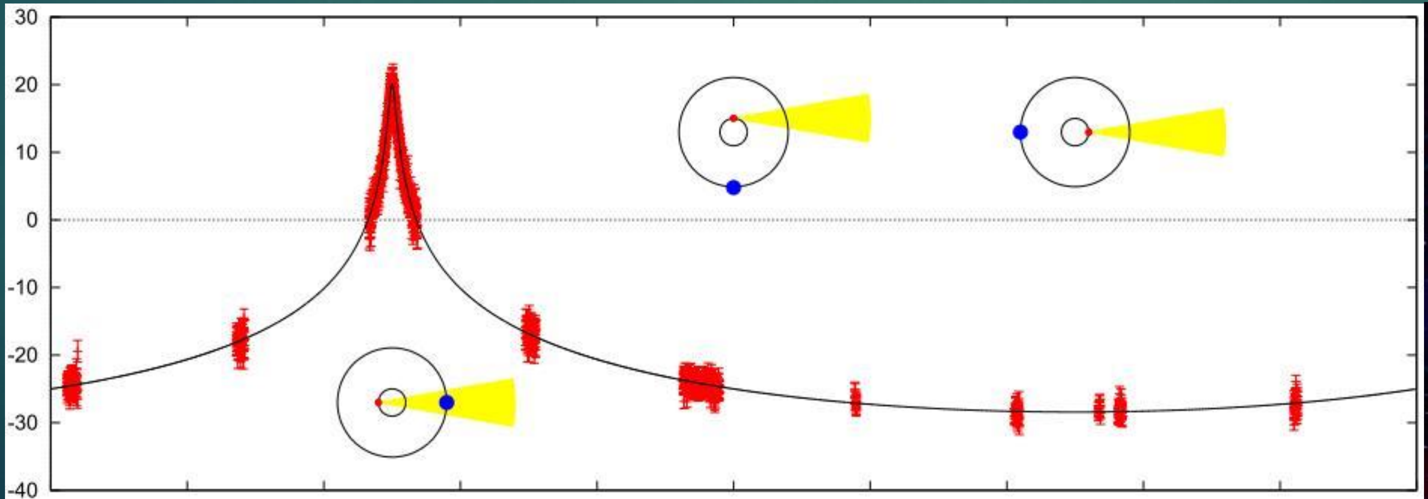


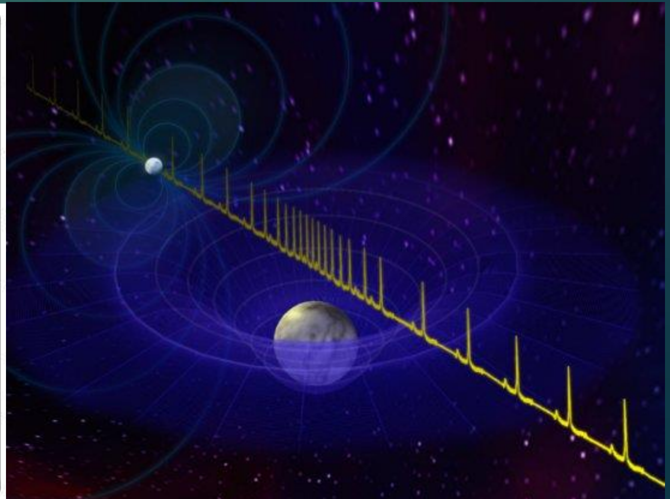
Neutron Star Data

- Data situation in general terms is good (masses, temperatures, ages, frequencies)
- Ability to explain the data with different models in general is good, too.

... which sounds good, but becomes tiresome if everybody explains everything …

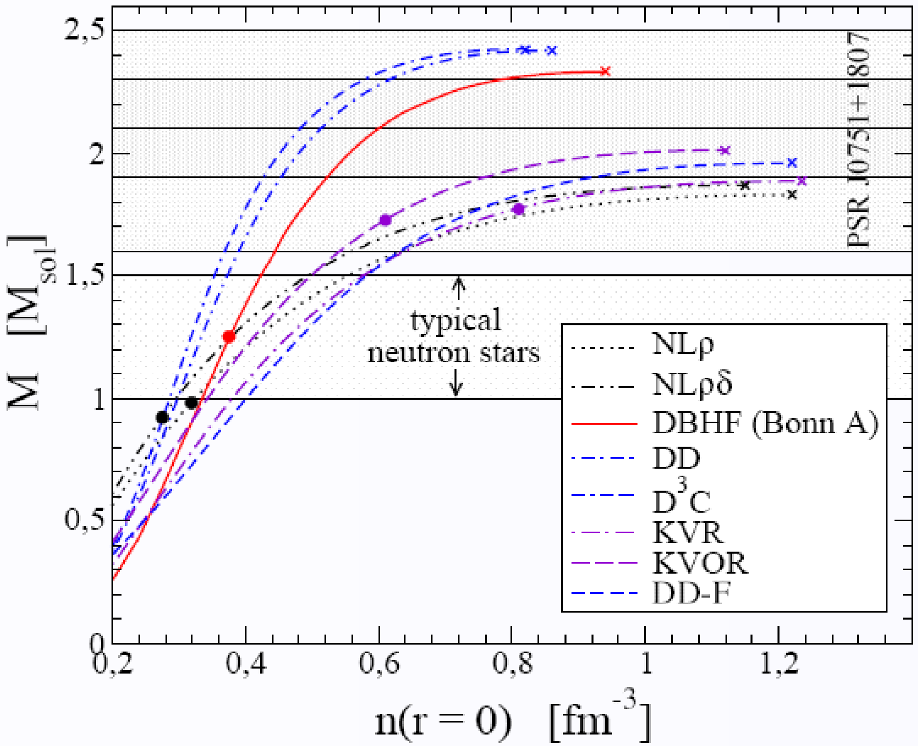
- For our purpose only a few observables are of real interest
- Most promising: High Massive NS with 2 solar masses (Demorest et al., Nature 467, 1081-1083 (2010))

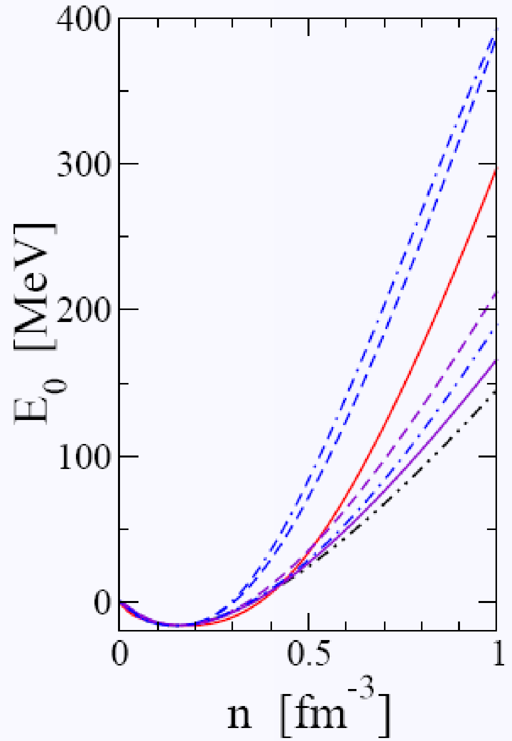




NS masses and the (QM) Equation of State

- **NS mass is sensitive mainly to the sym. EoS (In particular true for heavy NS)**
- Folcloric:
  QM is soft, hence no NS with QM core
- Fact:
  QM is softer, but able to support QM core in NS
- Problem:
  (transition from NM to) QM is barely understood





$M(n)$ correlated to $E_0(n)$

stiff: higher M_{max} at smaller densities

soft: smaller M_{max} at higher densities

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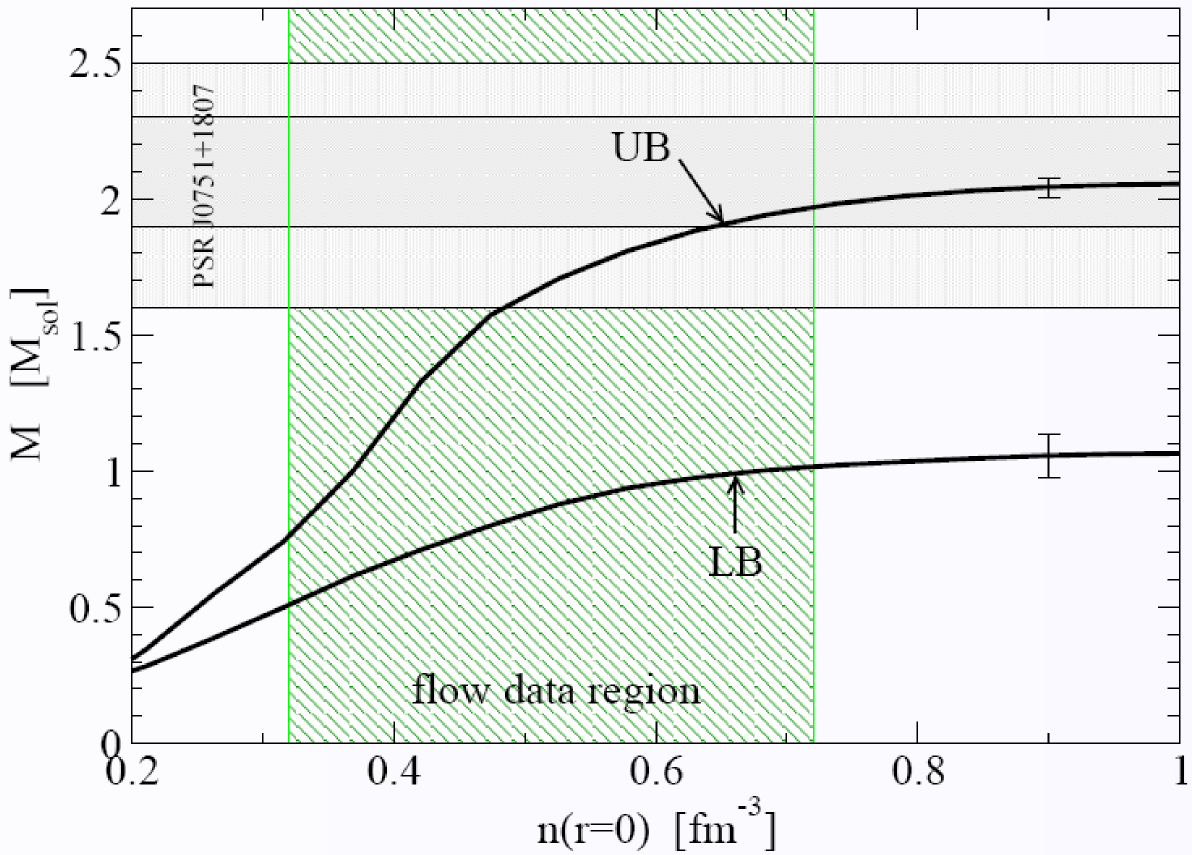

(applied “universal” $\beta^2 E_S$ (error bars!))

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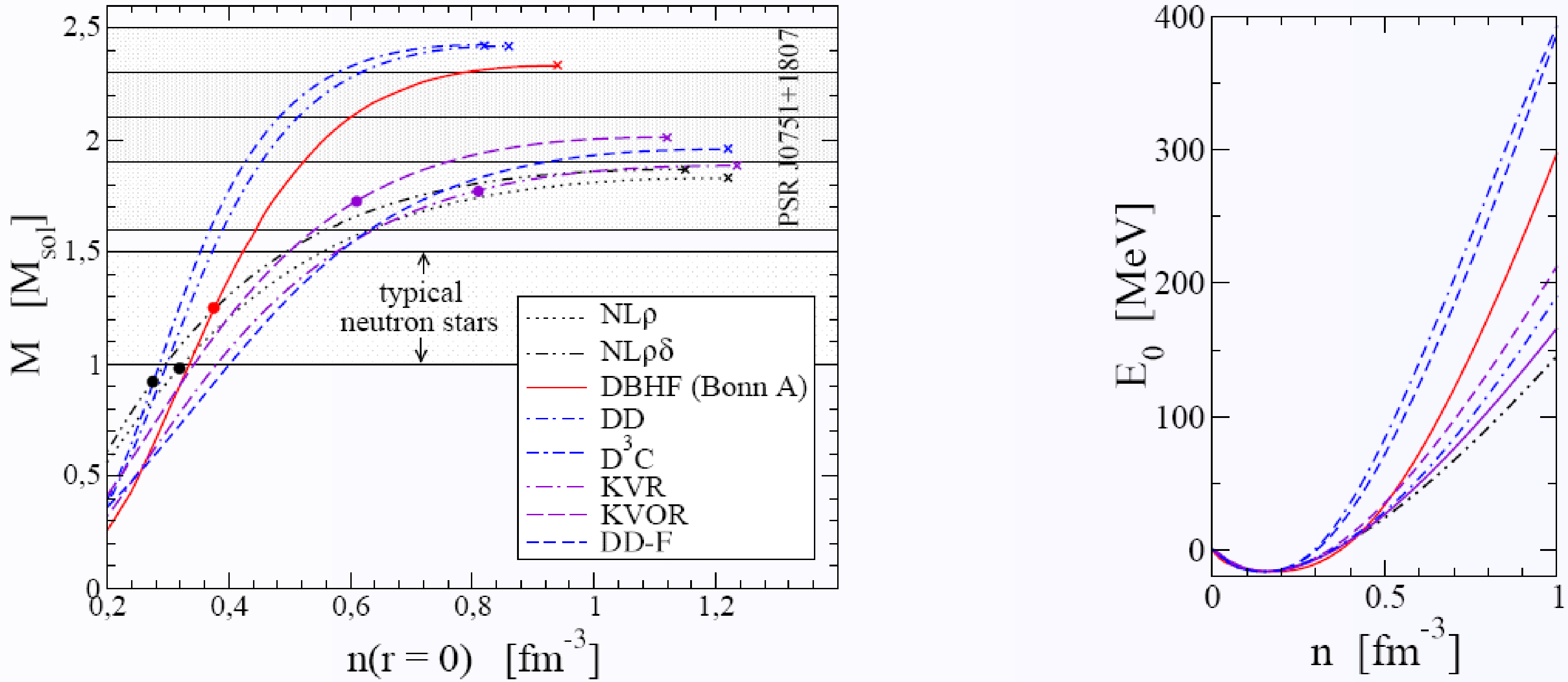

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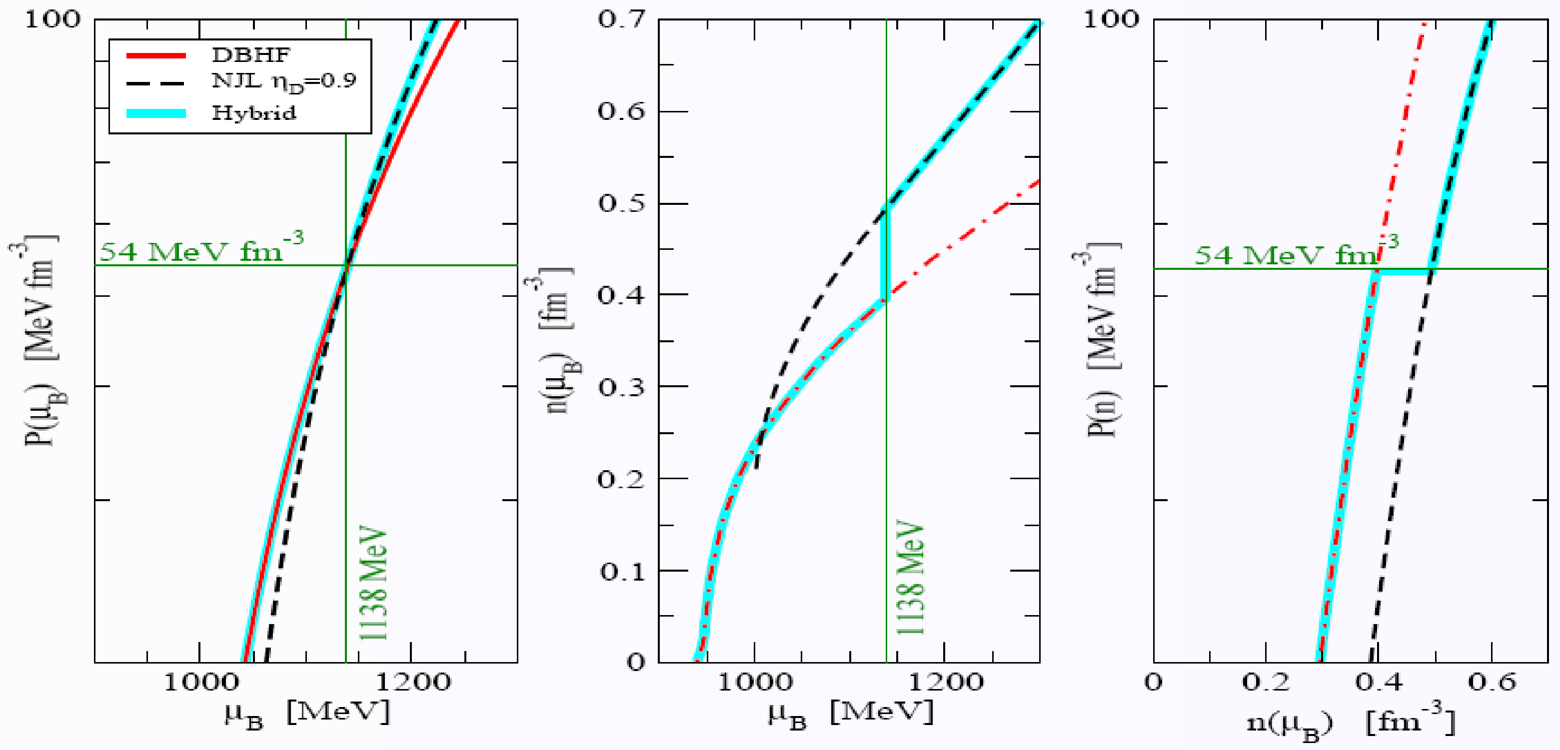
traditional: two-phase construction



"masquerade" problem: quark and hadron eos almost identical!

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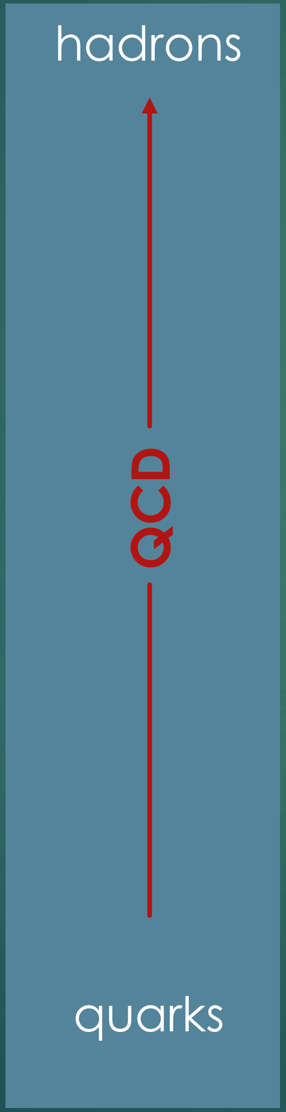
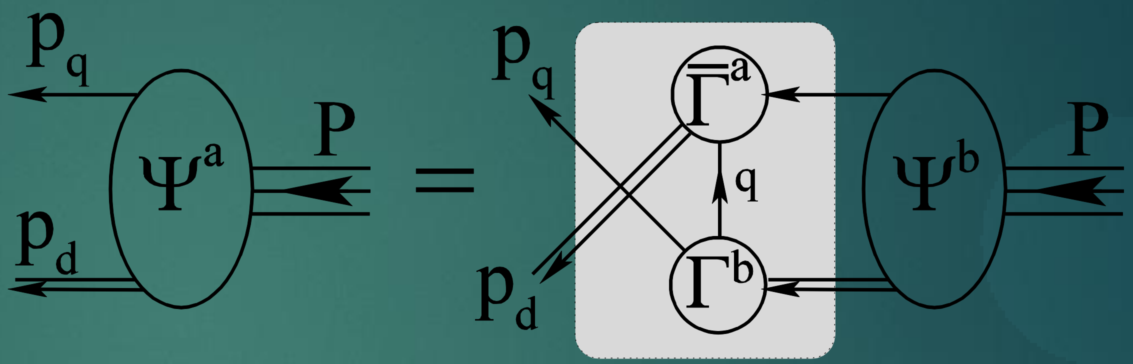


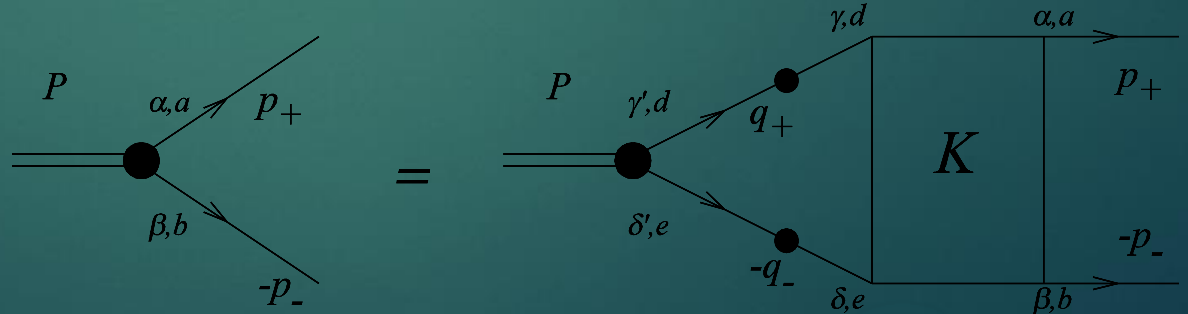
Dense Nuclear Matter in terms of Quark DoF is barely understood

<u>Problem is attacked in vacuum</u> **Faddeev Equations**



Baryons as composites of confined quarks and diquarks

Bethe Salpeter Equations



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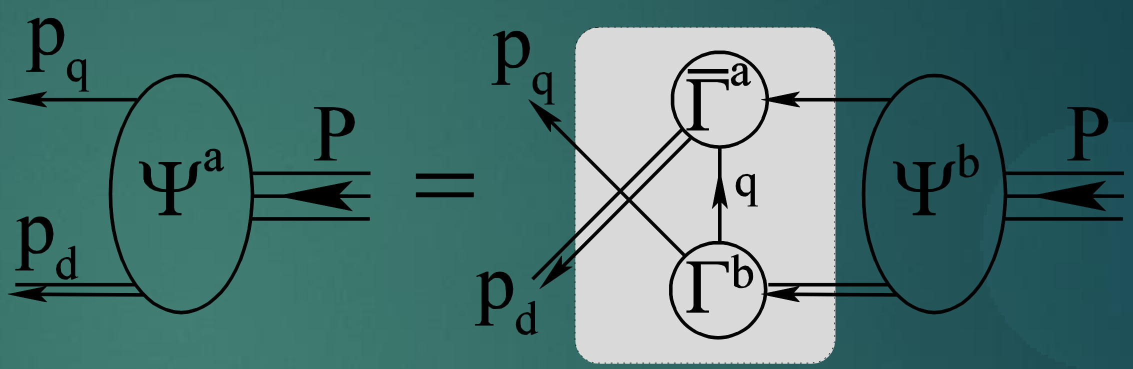

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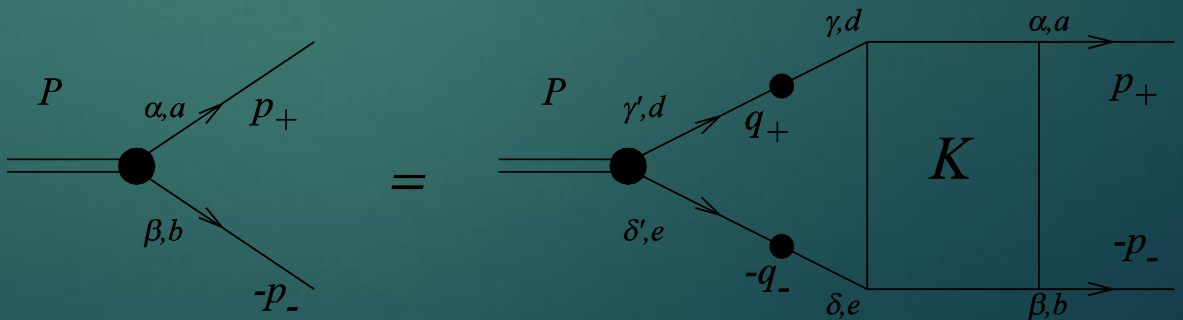


QCD in dense matter

- LQCD fails in dense (like DENSE) matter (Fermion-sign problem)
- Perturbative QCD fails in non-perturbative domain

  DCSB is explicitly not covered by perturbative approach:

$$B(p^2) = m\left(1 - \frac{\alpha_S}{\pi}\ln\left[\frac{p^2}{m^2}\right]\right) \quad \lim_{m_0 \to 0} \Sigma_S(p^2, m_0^2) = 0$$

- Solution: ‘some’ non-perturbative approach ‘as close as possible’ to QCD

  some = solvable; as close as possible = if possible DCSB, if possible confinement
- State of the art: Nambu-Jona-Lasinio model(s) (+bag models, +hybrids)

NJL type model

- S: DCSB
- V: renormalizes μ
- D: diquarks → 2SC, CFL
- TD Potential minimized
  in mean-field approximation
- Effective model by its nature; can be motivated (1g-exchange) doesn't have to though and can be extended (KMT, PNJL)
- possible to describe nucleons; not to be confused with confinement!

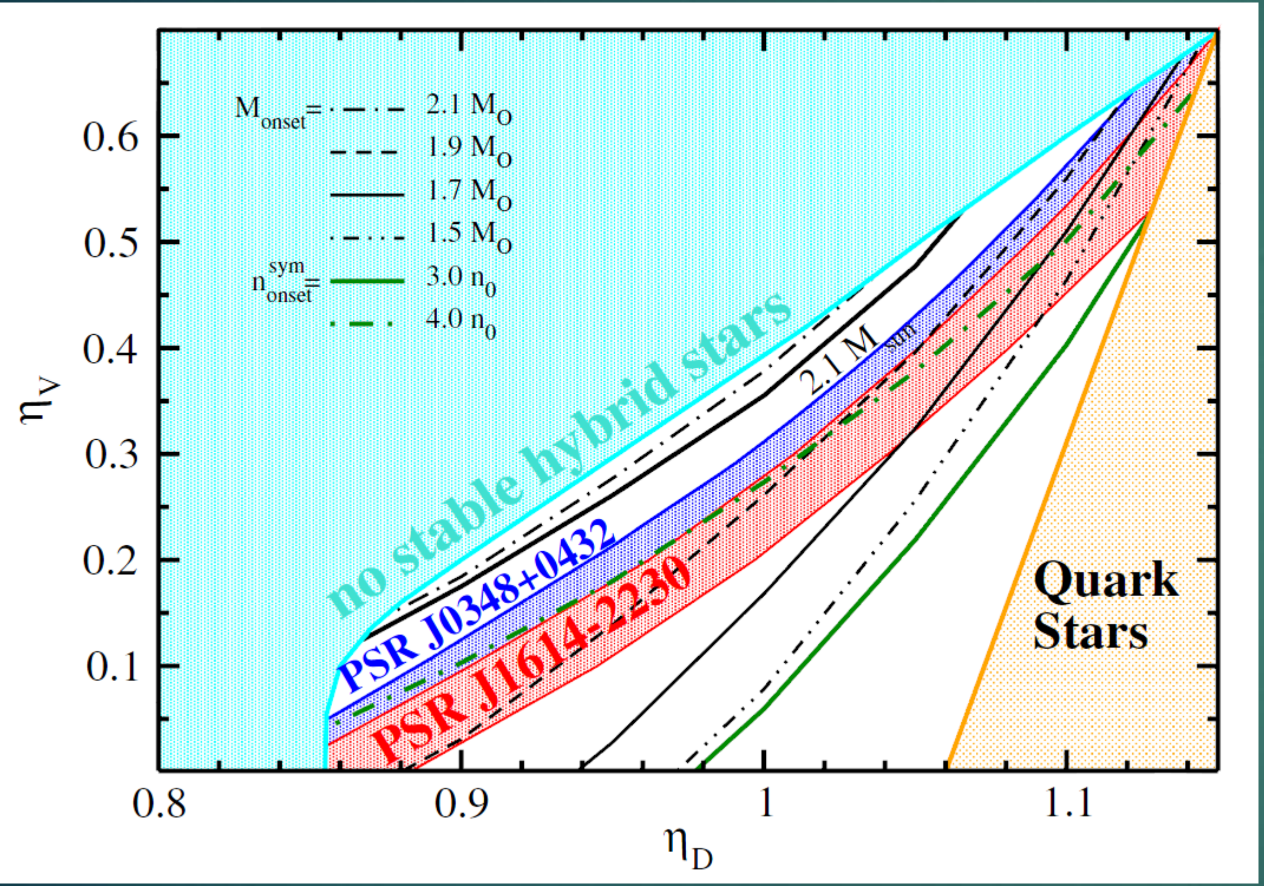
Effective Lagrangian

$$\mathcal{L}_{int} = G_S \eta_D \sum_{a,b=2,5,7} (\bar{q} i \gamma_5 \tau_a \lambda_b C \bar{q}^T)(q^T C i \gamma_5 \tau_a \lambda_a q) + G_S \sum_{a=0}^{8} \left[(\bar{q} \tau_a q)^2 + \eta_V (\bar{q} i \gamma_0 q)^2 \right]$$

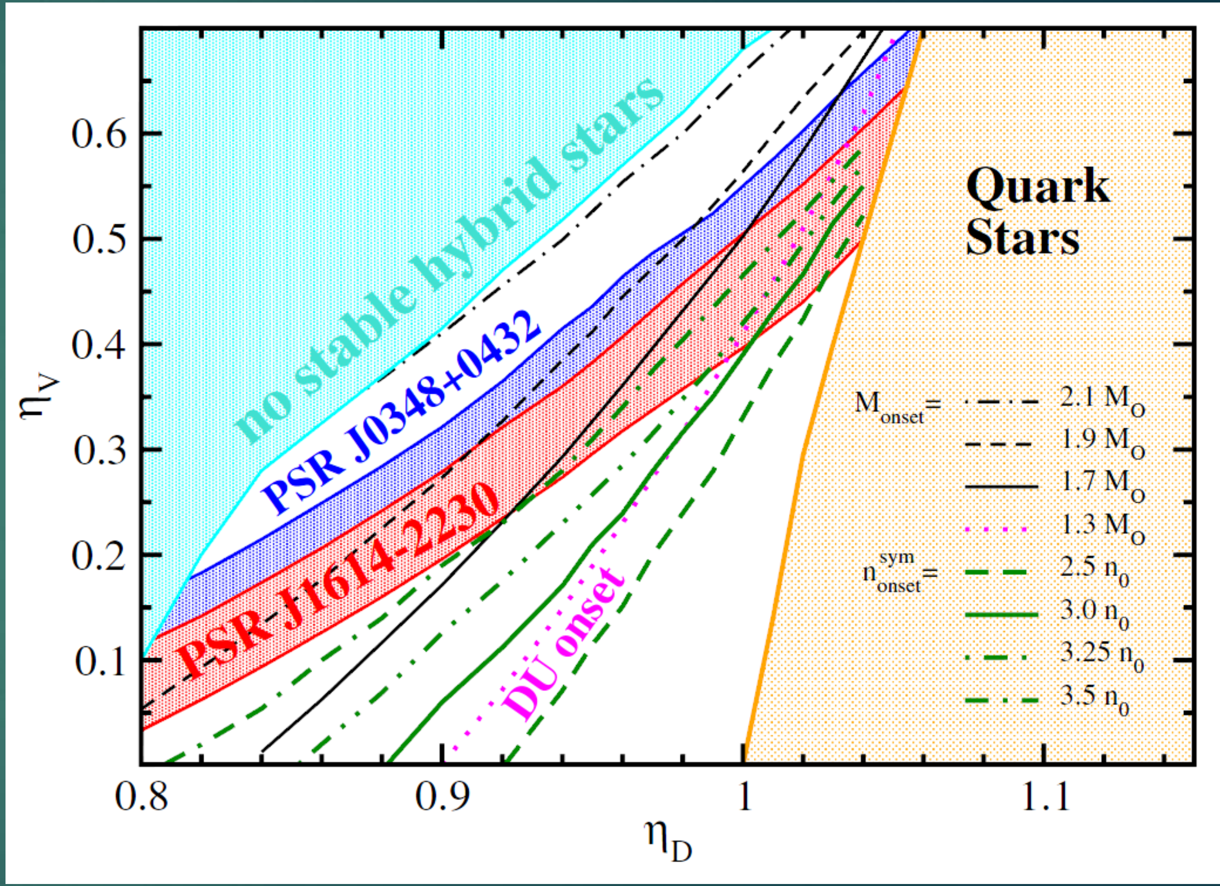
Thermodynamical potential

$$\Omega(T,\mu) = \frac{\phi_u^2 + \phi_d^2 + \phi_s^2}{8 G_S} + \frac{\Delta_{ud}^2 + \Delta_{us}^2 + \Delta_{ds}^2}{4 G_D} - \int \frac{d^3 p}{(2\pi)^3} \sum_{n=1}^{18} \left[E_n + 2 T ln \left(1 + e^{-E_n/T} \right) \right] + \Omega_{lep} - \Omega_0$$

NJL model study for NS (TK, R.Łastowiecki, D.Blaschke, PRD **88**, 085001 (2013))



Set A



Set B

Conclusion: NS may or may not support a significant QM core.
Other interaction channels won't change this if their coupling strengths are not precisely known.

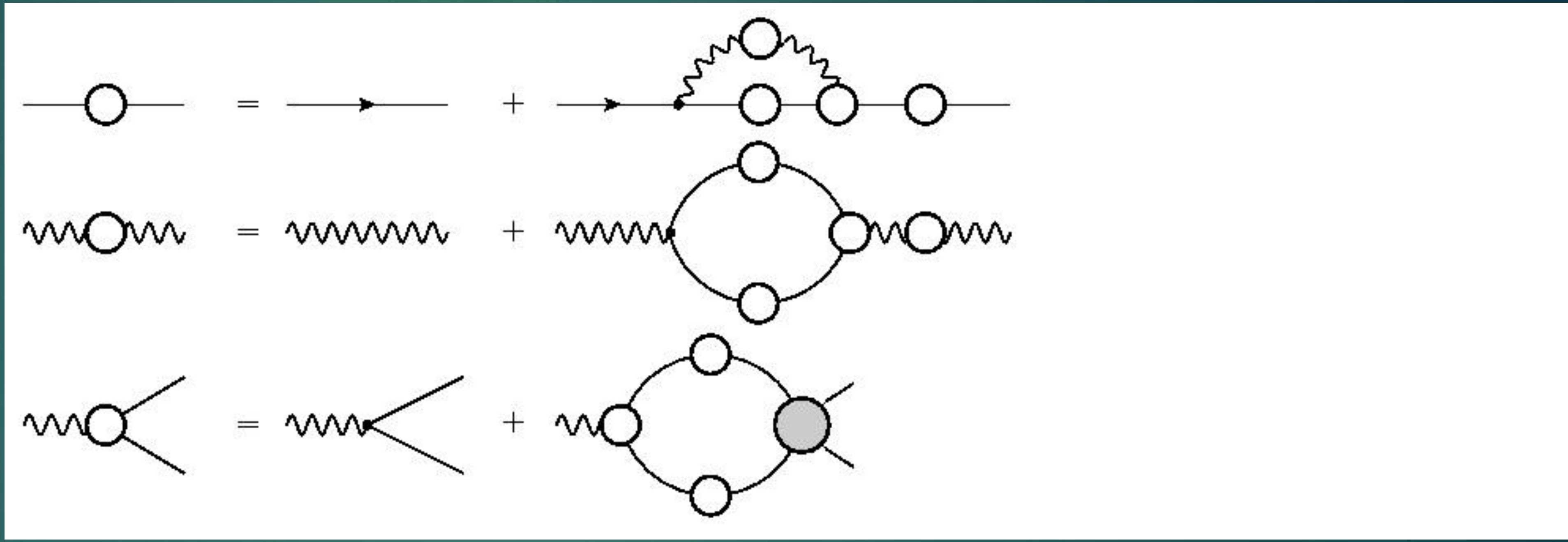
Beyond NJL

- NJL model can be understood as an approximate solution of Dyson-Schwinger equations

quark

gluon

q-g-vertex



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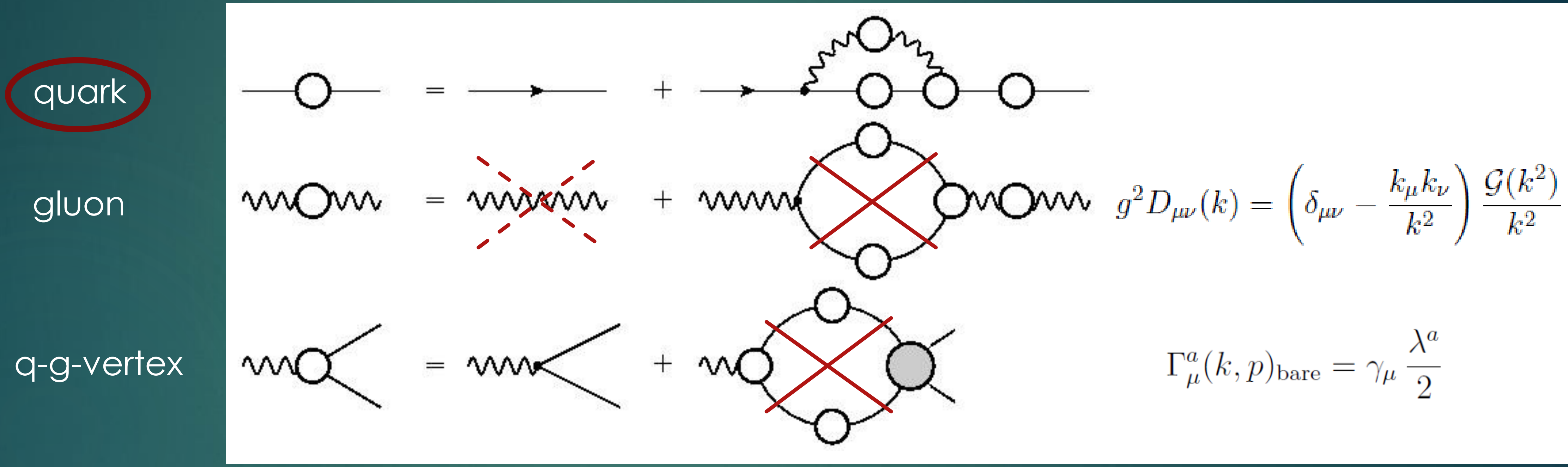

single particle: quark self energy

Inverse Quark Propagator:

$$S(p;\mu)^{-1} = Z_2(\underbrace{i\,\vec{\gamma}\,\vec{p} + i\,\gamma_4(p_4}_{= i\,\gamma\, p} + i\mu) + m_{\mathrm{bm}}) + \Sigma(p;\mu)$$

revokes Poincaré covariance

Renormalised Self Energy:

$$\Sigma(p;\mu) = Z_1 \int_q^\Lambda g^2(\mu) D_{\rho\sigma}(p-q;\mu) \frac{\lambda^a}{2} \gamma_\rho S(q;\mu) \Gamma_\sigma^a(q,p;\mu)$$

Loss of Poincaré covariance increases complexity

→ technically and numerically more challenging → no surprise, though

General Solution:

Vacuum: $\boxed{\mu = 0}$ $\quad S(p^2)^{-1} = i\,\gamma\, p\, A(p^2) + B(p^2)$

Medium: $\boxed{\mu \neq 0}$ $\quad S(p^2, p_4;\mu)^{-1} = i\,\vec{\gamma}\,\vec{p}\, A(p^2, p_4, \mu) + i\,\gamma_4(p_4 + i\mu)\, C(p^2, p_4, \mu) + B(p^2, p_4, \mu)$

Similar structured equations in vacuum and medium, but in medium:

1. one more gap
2. gaps are complex valued
3. gaps depend on (4-)momentum, energy and chemical potential

Effective gluon propagator

$$S(p;\mu)^{-1}=Z_2(i\,\vec{\gamma}\,\vec{p}+i\,\gamma_4(p_4+i\mu)+m_{\mathrm{bm}})+\Sigma(p;\mu)$$

$$\Sigma(p;\mu)=Z_1\int_q^\Lambda g^2(\mu)D_{\rho\sigma}(p-q;\mu)\frac{\lambda^a}{2}\gamma_\rho S(q;\mu)\Gamma_\sigma^a(q,p;\mu)$$

Ansatz for self energy (rainbow approximation, effective gluon propagator(s))

$$Z_1\int_q^\Lambda g^2 D_{\mu\nu}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\Gamma_\nu^a(q,p)\rightarrow\int_q^\Lambda \mathcal{G}((p-q)^2)\,D_{\mu\nu}^{\mathrm{free}}(p-q)\frac{\lambda^a}{2}\gamma_\mu S(q)\frac{\lambda^a}{2}\gamma_\nu$$

Specify behaviour $\mathcal{G}(k^2)$

$$\frac{\mathcal{G}(k^2)}{k^2}=8\pi^4 D\delta^4(k)+\frac{4\pi^2}{\omega^6}Dk^2\mathrm{e}^{-k^2/\omega^2}+4\pi\frac{\gamma_m\pi}{\frac{1}{2}\ln\left[\tau+\left(1+k^2/\Lambda_{\mathrm{QCD}}^2\right)^2\right]}\mathcal{F}(k^2)$$

Infrared strength (zero width + finite width contribution) — running coupling for large k

Results at finite densities obtained for

1st term (Munczek/Nemirowsky (1983)) → Klähn et al. (2010)

2nd term → Chen et al.(2008,2011)

NJL model: $g^2 D_{\rho\sigma}(p-q)\;=\;\frac{1}{m_G^2}\delta_{\rho\sigma}$ delta function in configuration(!) space

NJL model within DS framework

$$B(p) = m + \frac{16}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{B(q)}{\vec{q}^2A^2(q)+\tilde{q}_4^2C^2(q)+B^2(q)},$$

$$\vec{p}^2A(p) = \vec{p}^2 + \frac{8}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{\vec{p}\vec{q}A(q)}{\vec{q}^2A^2(q)+\tilde{q}_4^2C^2(q)+B^2(q)},$$

$$\tilde{p}_4^2C(p) = \tilde{p}_4^2 + \frac{8}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{\tilde{p}_4\tilde{q}_4C(q)}{\vec{q}^2A^2(q)+\tilde{q}_4^2C^2(q)+B^2(q)}.$$

To satisfy these equations all gap solutions have to be momentum independent. Simplest solution: A=1

$$\frac{8}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{\tilde{q}_4C(q)}{\vec{q}^2+\tilde{q}_4^2C^2(q)+B^2(q)} = iK$$

$$\tilde{p}_4^2C = \tilde{p}_4^2 + i\tilde{p}_4K$$

$$\Rightarrow \tilde{p}_4C = p_4 + i(\mu+K)$$

Renormalization of chem. pot. due to vector interaction

$$B = m + \frac{16}{3m_G^2}\int \frac{d^4q}{(2\pi)^4}\frac{B}{\vec{q}^2+\hat{q}_4^2+B^2}$$

mass gap equation

This is a 1 to 1 reproduction of the (basic) NJL model

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NJL model within DS framework

$$P[S] = \mathrm{Tr}\ln[S^{-1}] - \frac{1}{2}\mathrm{Tr}[\Sigma S]$$

Steepest descent approximation

$$P(\mu) = \int \frac{\mathrm{d}^4 p}{(2\pi)^4} \mathrm{Tr}\ln S^{-1}(\vec{p}^2, \hat{p}_4) + \frac{3}{4} m_G^2 K^2 - \frac{3}{8} m_G^2 B^2$$

1 to 1 NJL (regularization issue ignored)

$$\frac{8}{3m_G^2} \int \frac{d^4 q}{(2\pi)^4} \frac{\tilde{q}_4 C(q)}{\vec{q}^2 + \tilde{q}_4^2 C^2(q) + B^2(q)} = iK$$

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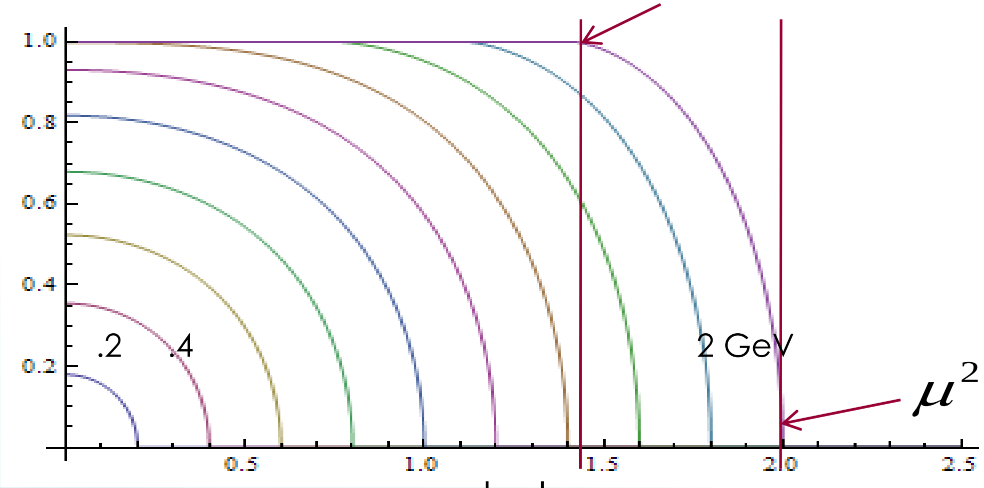
Model 1 (Munczek/Nemirowsky)

$$f_1(|\vec{p}|;\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\, \mathrm{tr_D}(-\gamma_4)S(p;\mu)$$

$$P(\mu<\eta) = P_0 + \int_0^\mu d\mu'\, n(\mu') \propto P_0 + const \times \mu^5$$

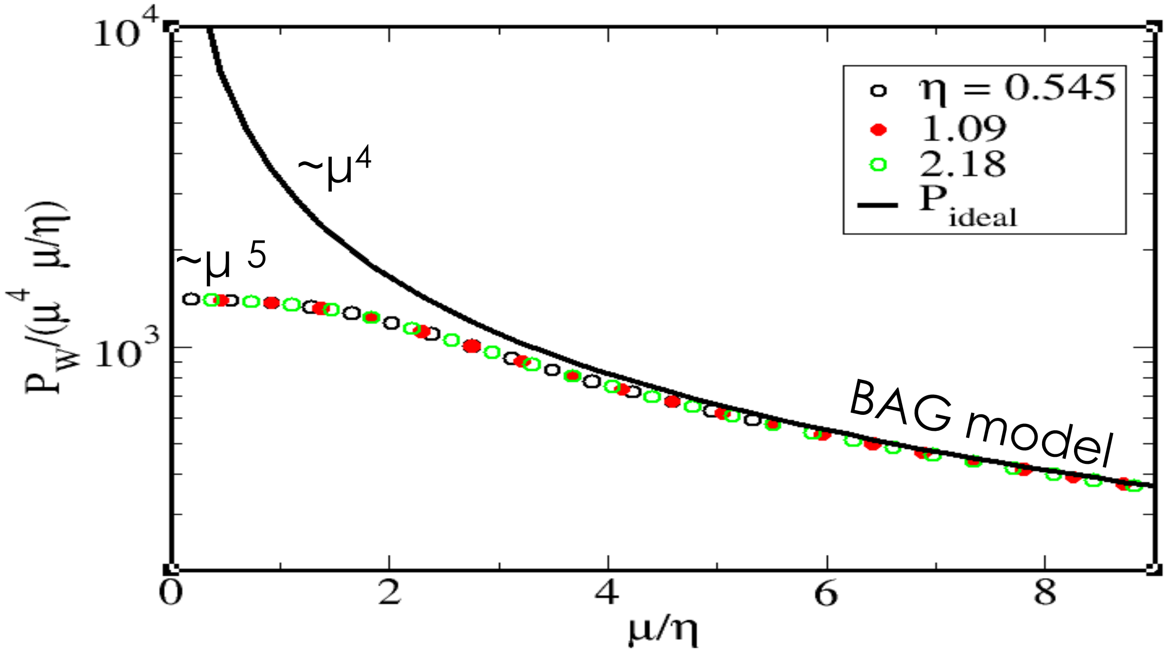
Wigner Phase

$\vec{p}^2 = \mu^2 - 2\eta^2$



μ^2

$|\vec{p}|$



$\mu^2 \geq 2\eta^2$ to obtain $f_1(\vec{p}^2 = 0) = 1$ model is scale invariant regarding μ/η

$P(\mu) \propto \mu^5$ well satisfied up to $\mu/\eta \approx 1$

$(\eta = 1.09\,\mathrm{GeV})$

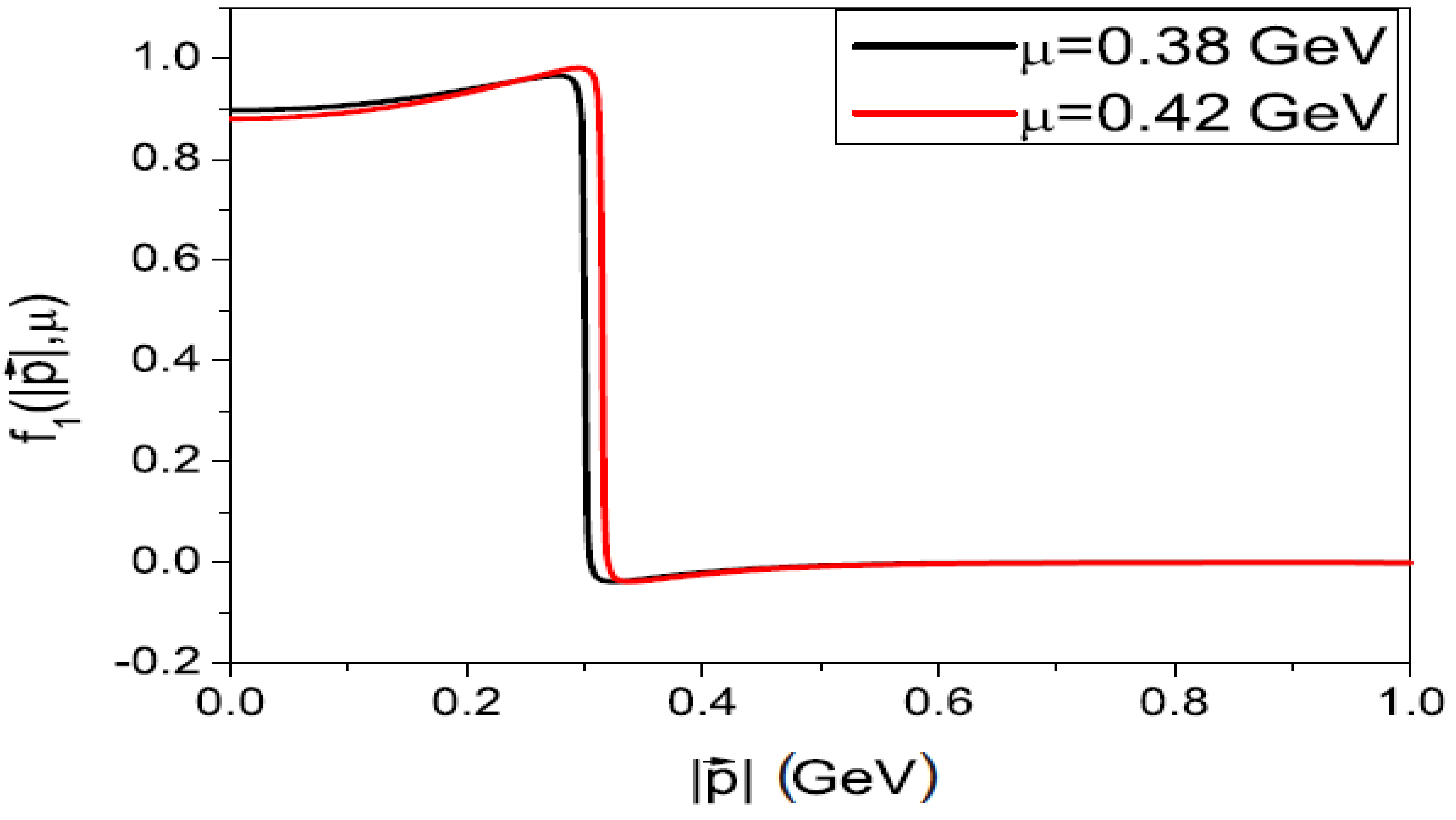
‚small‘ chem. Potential: $f_1(\vec{p}^2 = 0, \mu < \eta) \propto \mu \leftarrow$ $n(\mu<\eta) = \frac{2N_c N_f}{2\pi^2}\int d^3\vec{p}\ f_1(|\vec{p}|) \propto \mu^4$

T. Klahn, C.D. Roberts, L. Chang, H. Chen, Y.-X. Liu PRC 82, 035801 (2010)

Model 2

$$f_1(|\vec{p}|;\mu) = \frac{1}{4\pi}\int_{-\infty}^{\infty} dp_4\, \mathrm{tr}_D(-\gamma_4)S(p;\mu)$$

<u>Wigner Phase</u> Less extreme, but again, 1particle number density distribution different from free Fermi gas distribution



Chen et al. (TK) PRD 78 (2008)

Conclusions

NJL model is a powerful tool to explore possible features of dense QCD

It possibly might be a too powerful tool for unambiguous predictions

NJL mf approximation is a gluon mf approximation in DSE
which causes the known regularisation issues that could be avoided

Accounting for momentum dependent gap solutions enriches
the model space significantly – DSE successful in vacuum (hadron properties)

NB: Momentum independent gap solutions in their very nature
result in a quasi particle picture $\rightarrow$ no confinement