Understanding the nucleon's structure

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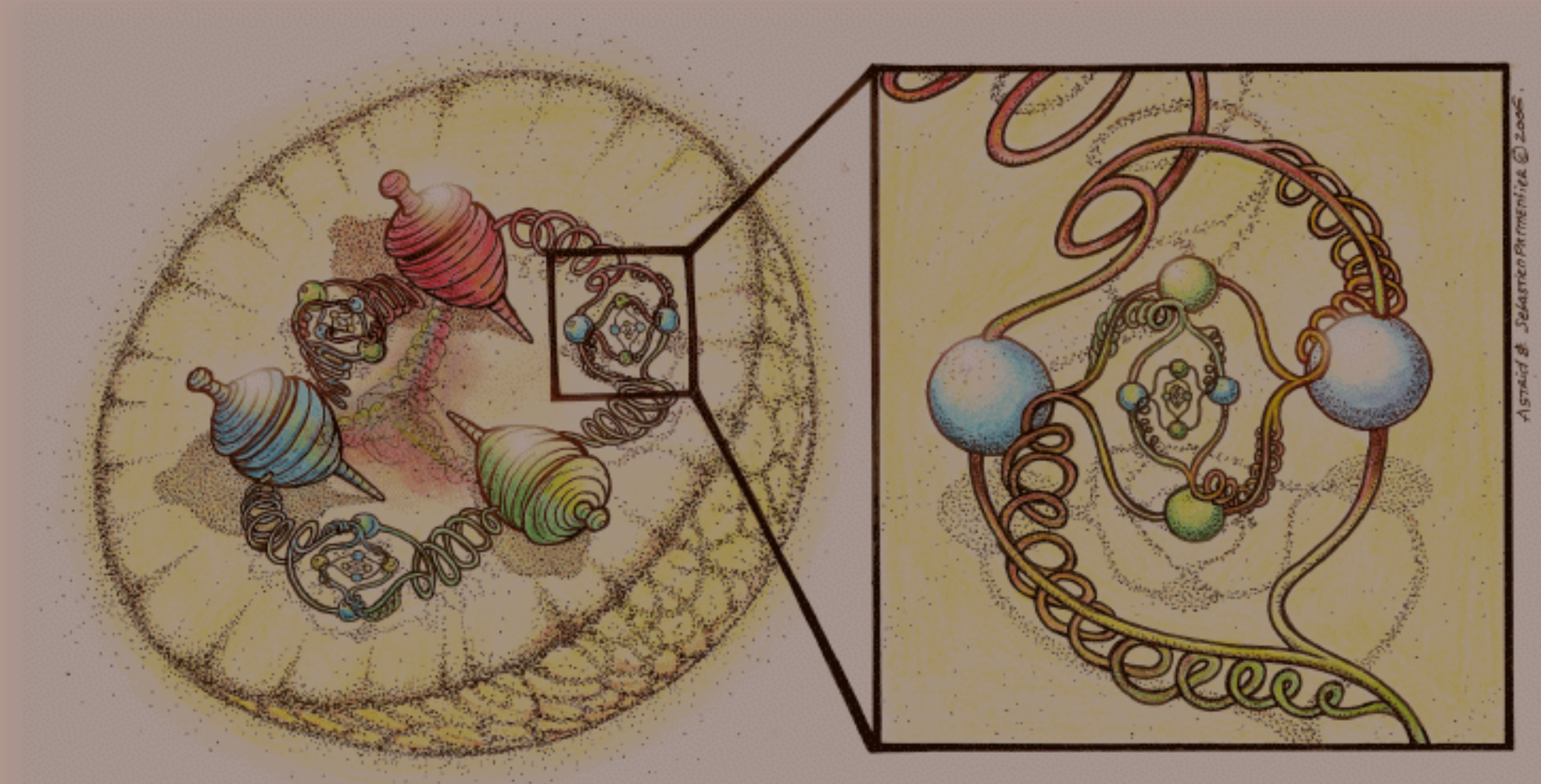


The QCD picture of the Proton Color Pencil and pen Drawing by Sebastien Parmentier and Astrid Morreale

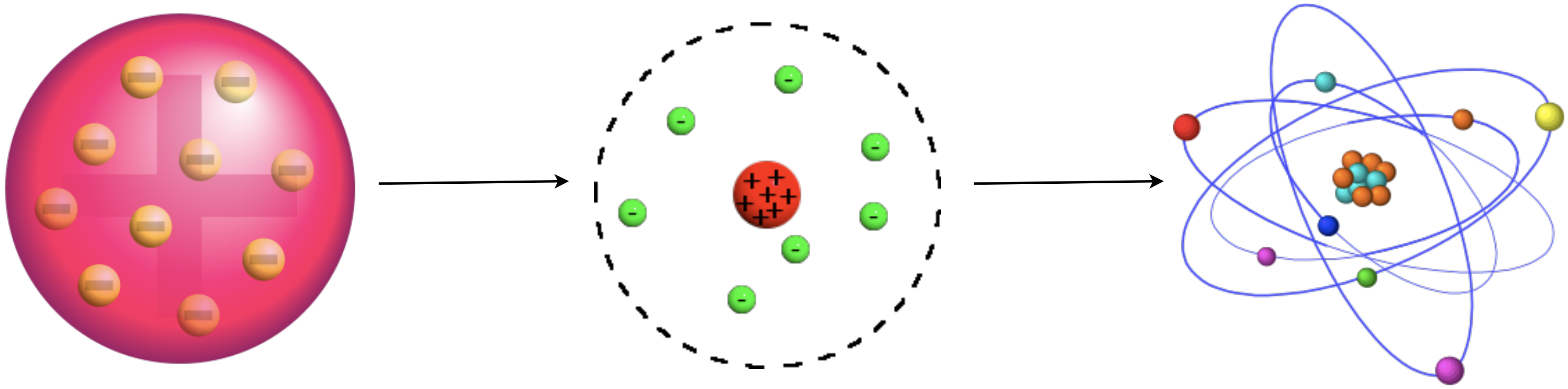
Outline

- Introduction & a bit of history
- 1-D nucleon structure: QPM and some unsolved problems
- Spin structure of the nucleon - quark and gluon's polarisation
- TMDs (3-D imaging), nucleon "tomography" (1+2-D imaging)
- Lattice QCD results
- Summary

Why do we want to know nucleon structure?

- Structure often leads to the revolution in knowledge
- Know what we are made of

example: Atomic structure (100 y.ago)



J.J. Thomson's
plum-pudding model

Rutherford's
planetary model

Modern model
Quantum orbitals

Discovery of nucleus
A localized charge/force center
A vast "open" space

Discovery of
Quantum Mechanics,
and
the Quantum World!

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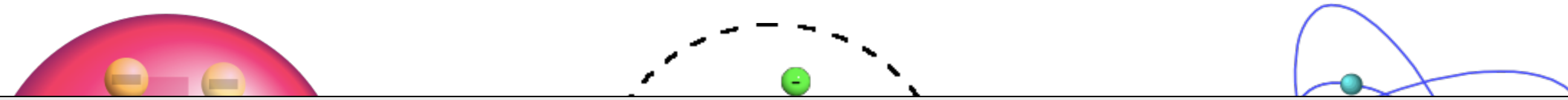


Completely changed our view of the visible world:
- Mass by “tiny” nuclei – less than 1 trillionth in volume
- Motion by quantum probability – the quantum world!

Provided infinite opportunities to improve things around us:
- Nano materials, quantum computing, …

Discovery of

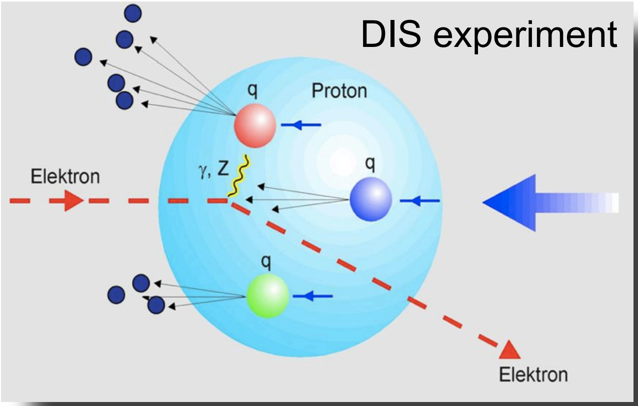
- Nucleon is not an elementary object;
  magnetic moments:

$$g_{proton} \neq 2 \qquad g_{neutron} \neq 0$$

How to "see" nucleon's structure?

Discovery of spin 1/2 quarks: SLAC

The birth of QCD (1973)
Quark Model + Yang-Mill gauge theory



Nobel Prize, 1990

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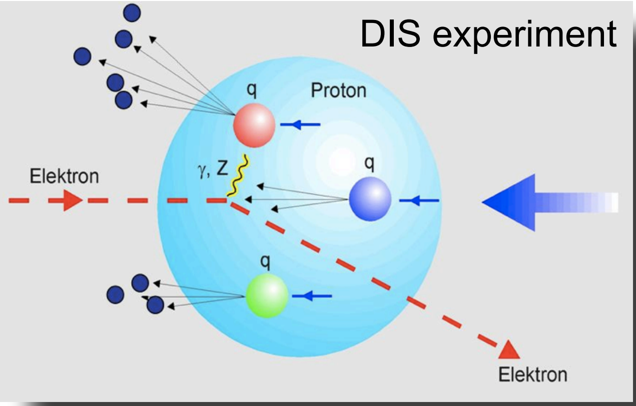


The birth of QCD (1973)
Quark Model + Yang-Mill gauge theory

Nobel Prize, 1990

Nucleon structure is complicate

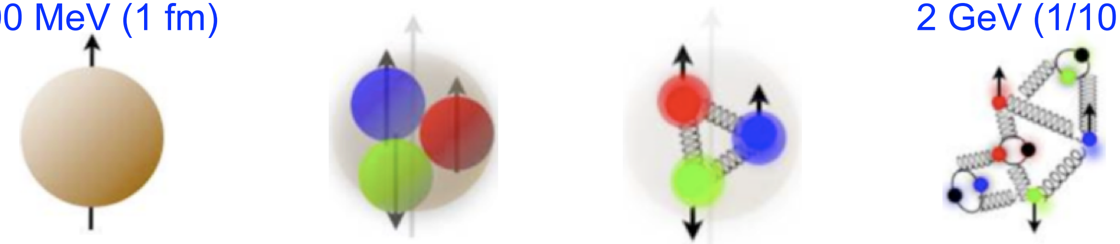
Color Confinement

Asymptotic freedom

Probing momentum

200 MeV (1 fm)

2 GeV (1/10 fm)

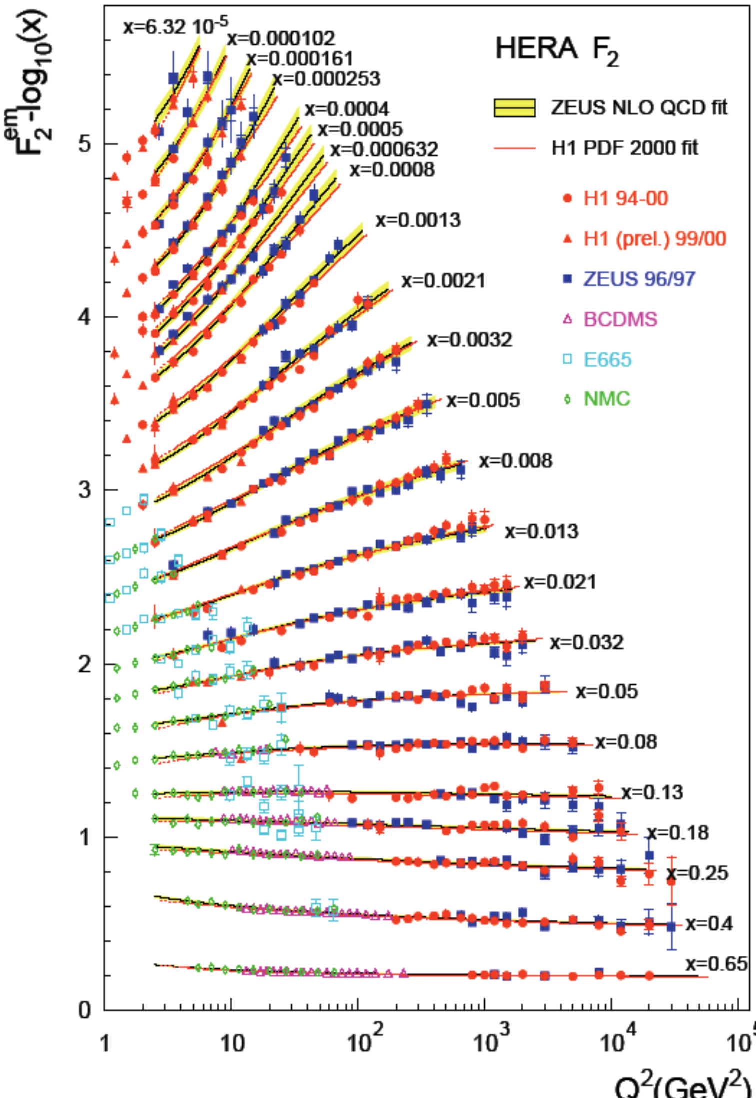


Nobel Prize, 2004

QCD

QCD@high energy:

Asymptotic freedom
+ perturbative QCD



QCD@high energy:

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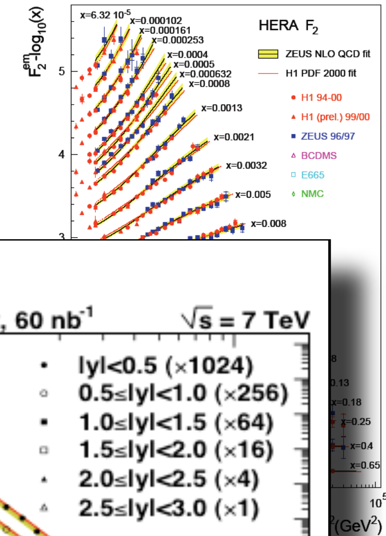

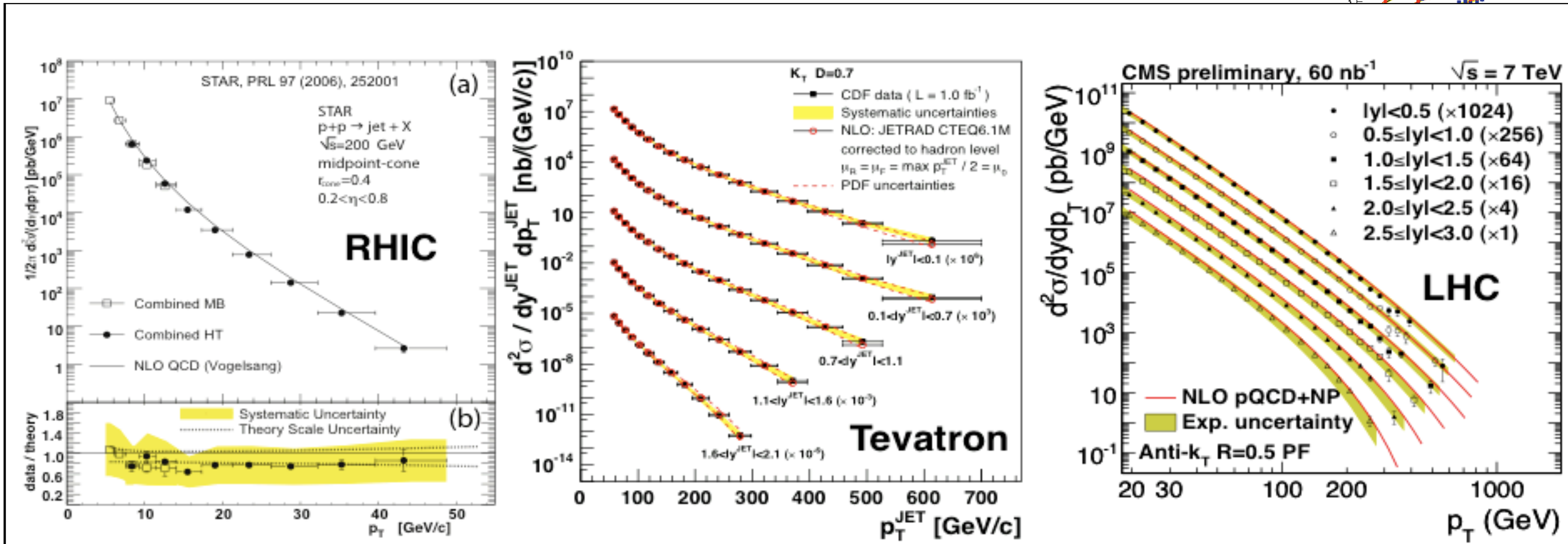

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Asymptotic freedom
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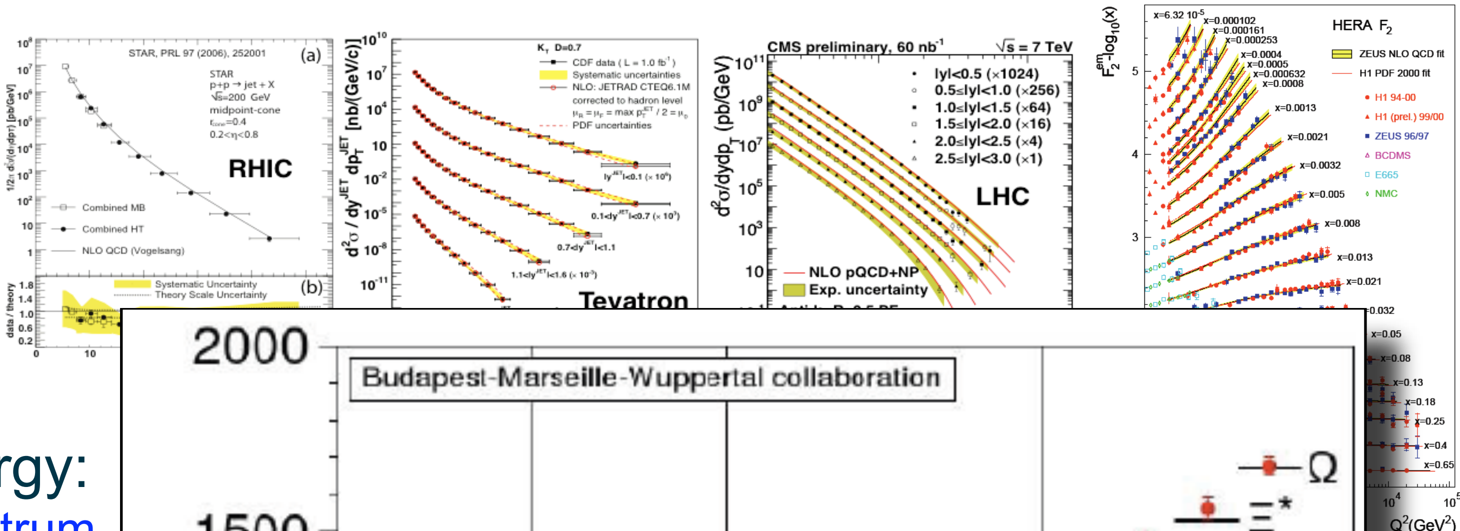
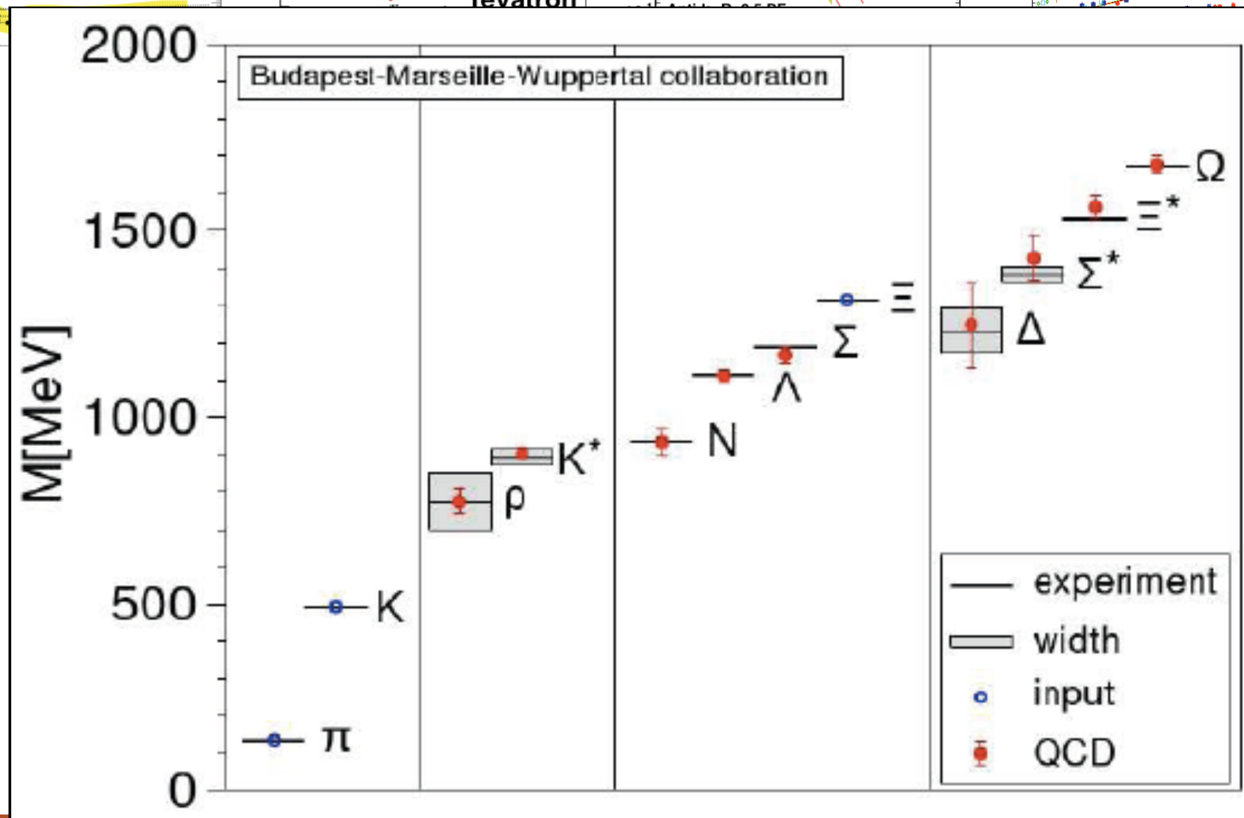


QCD@low energy:
Hadron mass spectrum
from lattice QCD



QCD

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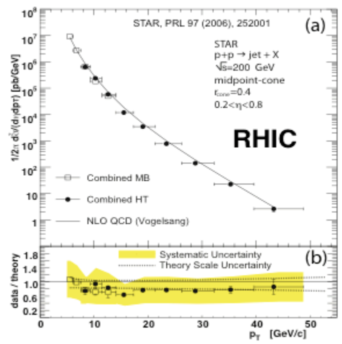
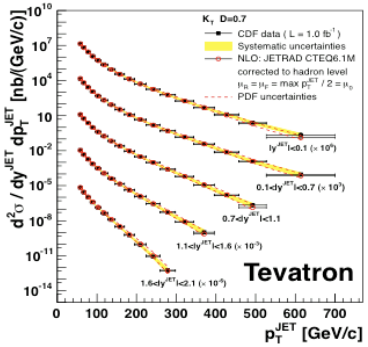
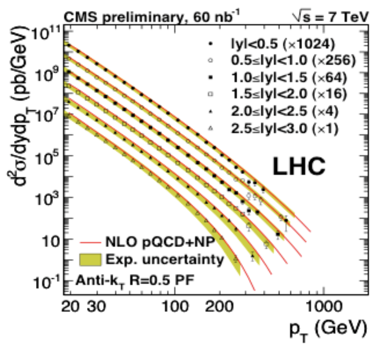
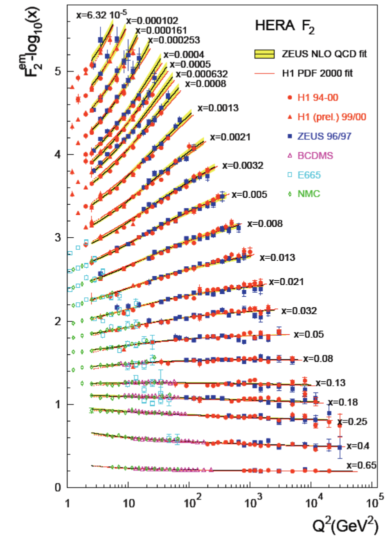




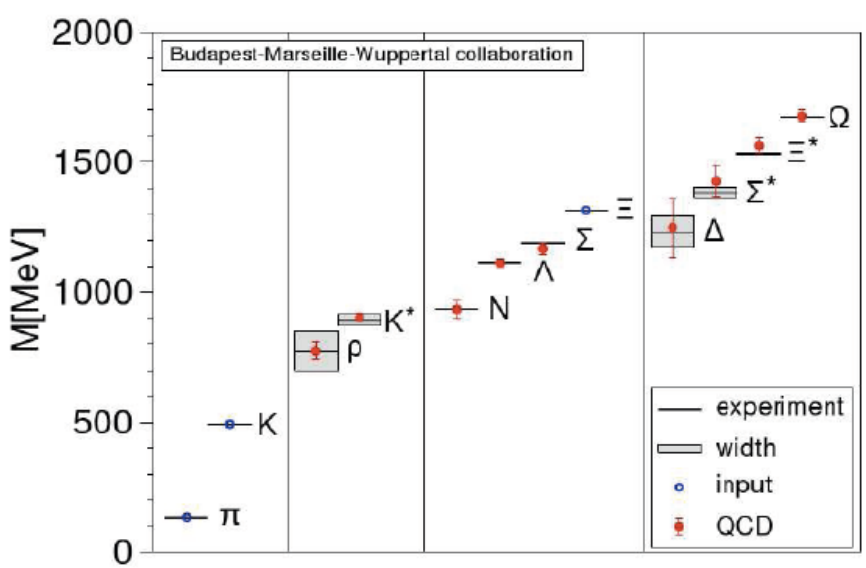




Measure e-p at 0.3 TeV (HERA)
Predict p-p and p-p at 0.2, 1.96, and 7 TeV

QCD@low energy:

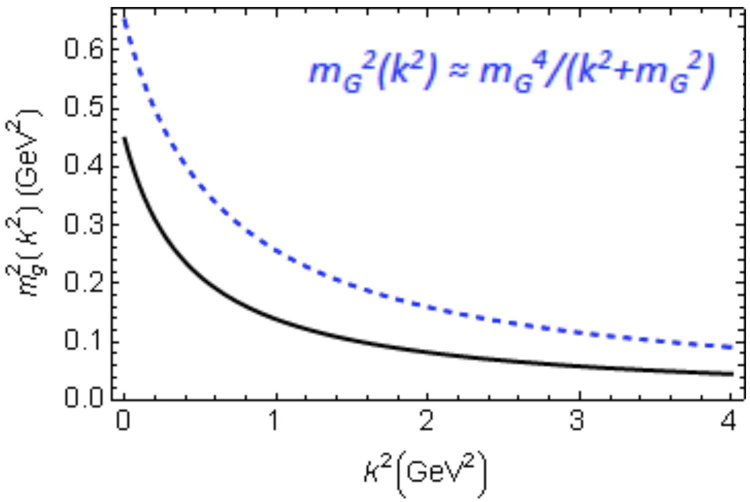
Hadron mass spectrum
from lattice QCD



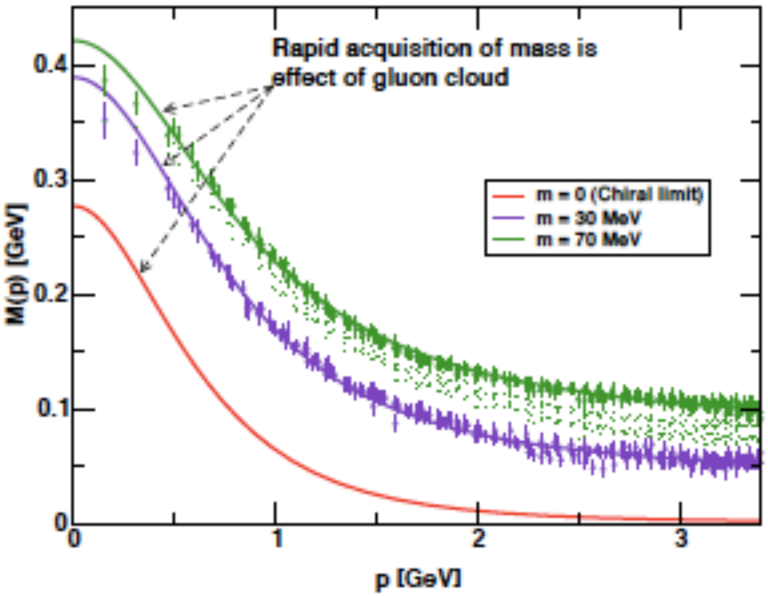
The hadron mass puzzle

$m_q \sim 10$ MeV $m_N \sim 1000$ MeV ⟶ Quarks carry ~ 1% nucleon's mass

How does QCD generate energy for the nucleon's mass?



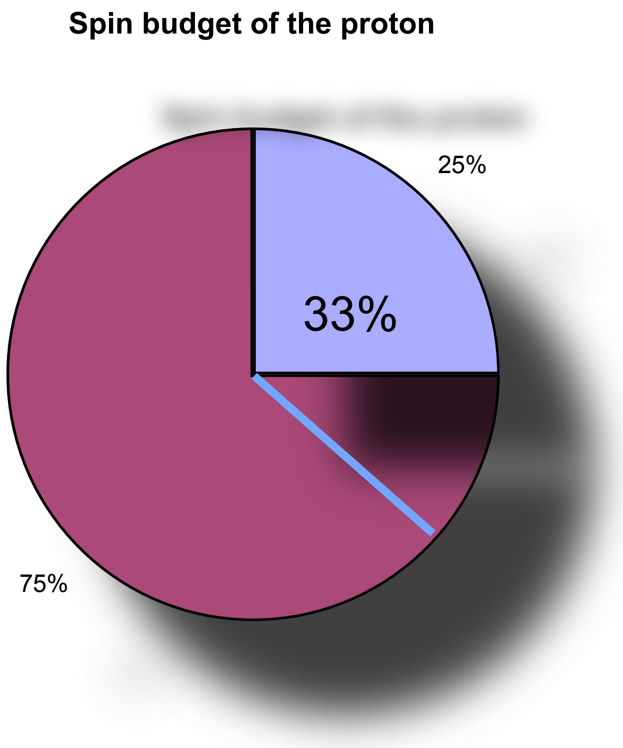
Qin et al., Phys. Rev. C 84 042202 (Rapid Comm.)



Mass from nothing!

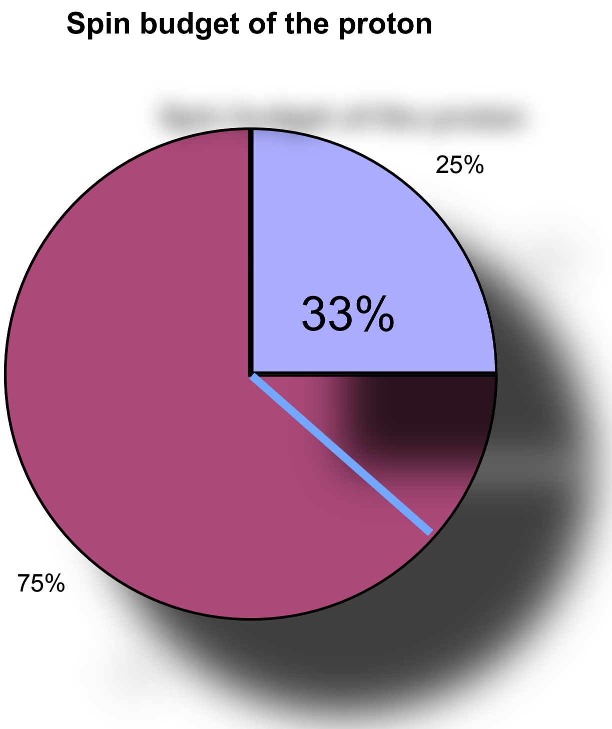
Light-quark mass comes from a cloud of soft gluons
Gluon is massless in UV, but "massive" in IR

The nucleon spin puzzle



"Dark" angular momentum

The nucleon spin puzzle



"Dark" angular momentum

$$a_{0|Q_0^2=3(GeV/c)^2} = 0.35 \pm 0.03(stat) \pm 0.05(syst)$$

QCD NLO

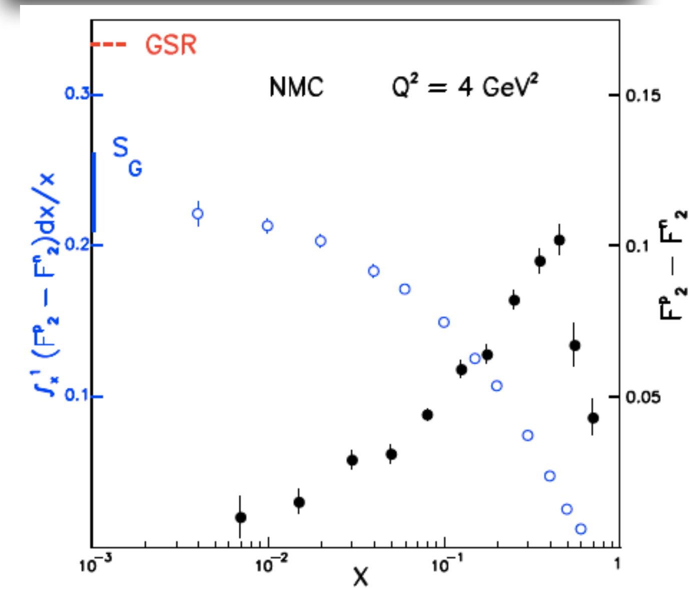
$$\hat{a}_{0|Q^2\to\infty} = 0.33 \pm 0.03(stat) \pm 0.05(syst)$$

beyond NLO

The driving question for QCD spin physics is where the nucleon spin comes from?

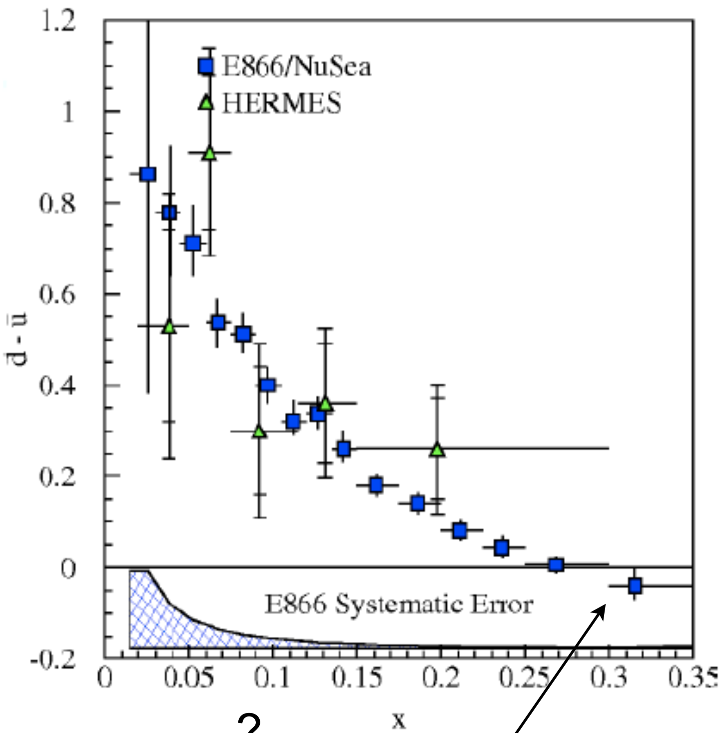
Violation of Gottfried sum rule

$$S_G = \int_0^1 [(F_2^p(x) - F_2^n(x))/x]dx$$
$$= \frac{1}{3} + \frac{2}{3}\int_0^1 (\overline{u}_p(x) - \overline{d}_p(x))\ dx$$
$$= \frac{1}{3} \quad (if\ \overline{u}_p = \overline{d}_p)$$



$$\text{NMC: } S_G = 0.235 \pm 0.026$$

Confirmed by Drell-Yan exp't



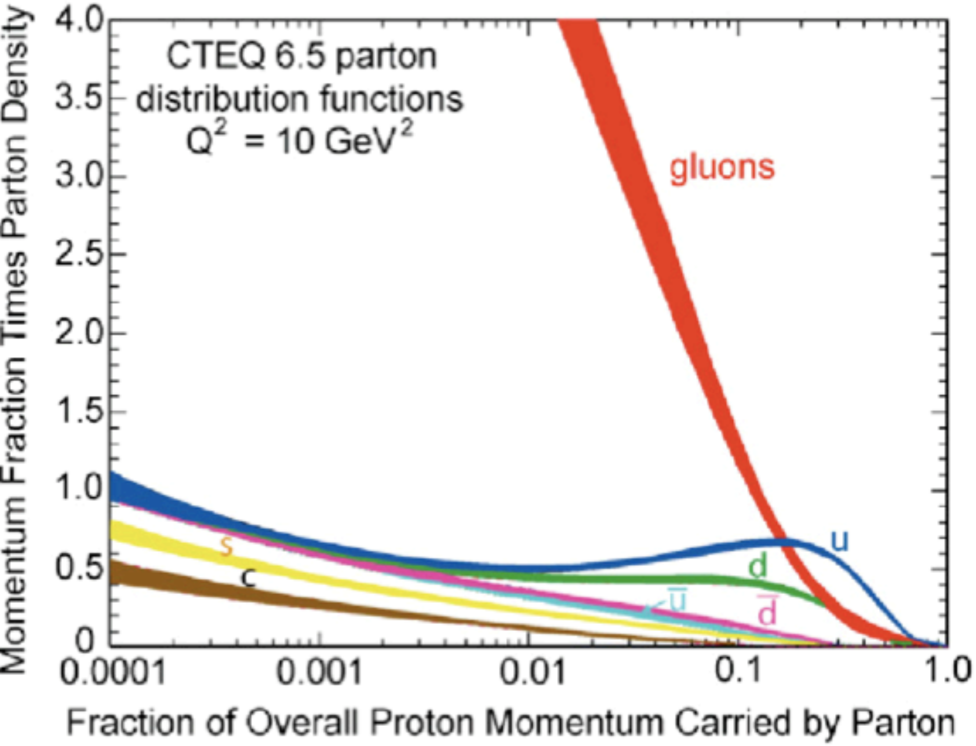
?

$$\bar{d}(x) \neq \bar{u}(x)$$

$\bar{d}(x) - \bar{u}(x)$ change sign?

All known models (Meson cloud, Chiral-quark soliton model, Statistic model) predict no sign change

Saturation phenomena

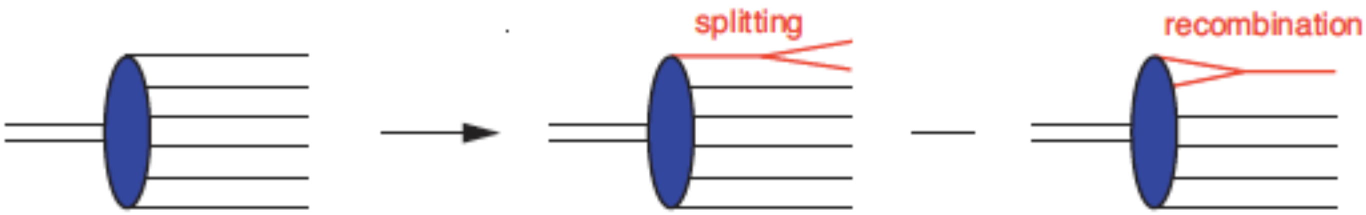


How does the unitarity bound of hadronic cross section survive if soft gluons in a proton or nucleus continue to grow in numbers?

Gluons interact among themselves when occupation number near 1

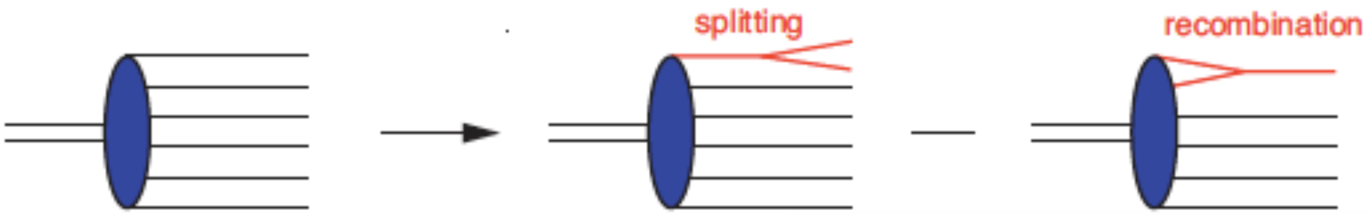
Instead of reaching Bose-Einstein condensate, gluon density saturates – a dynamical balance of non-linear QCD interaction

Saturation phenomena



Recombination of gluons leads to non linear effects – BK/JIMWLK evolution and phenomenon of saturation.

Saturation phenomena



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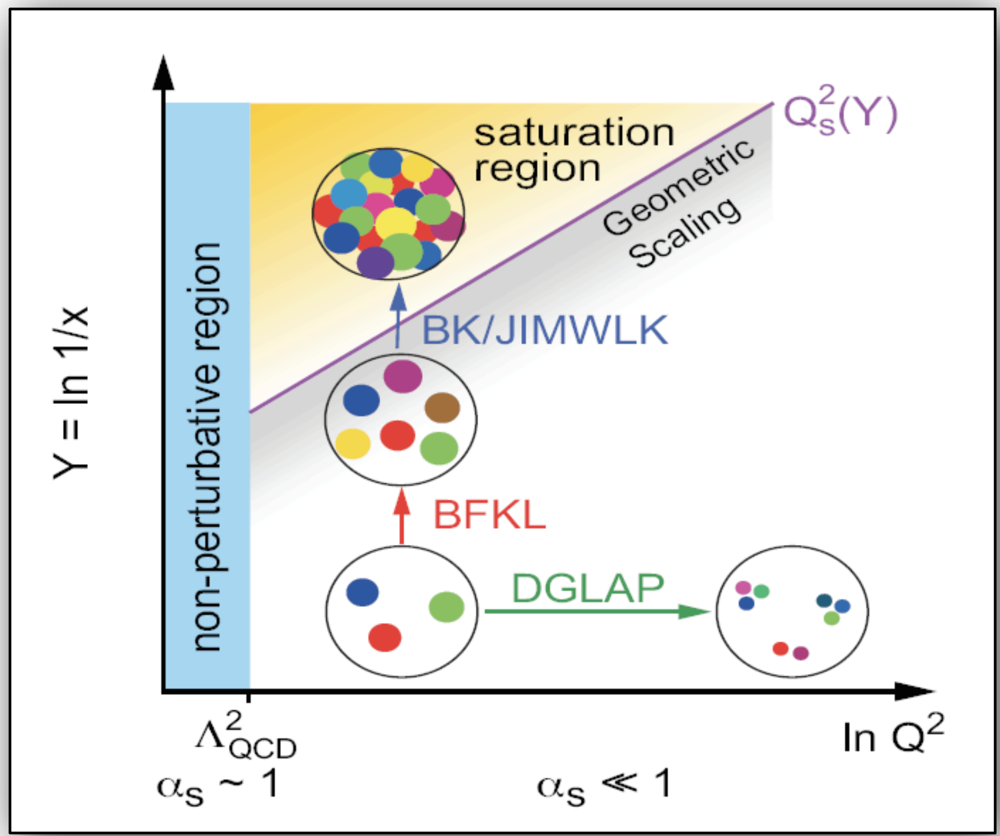
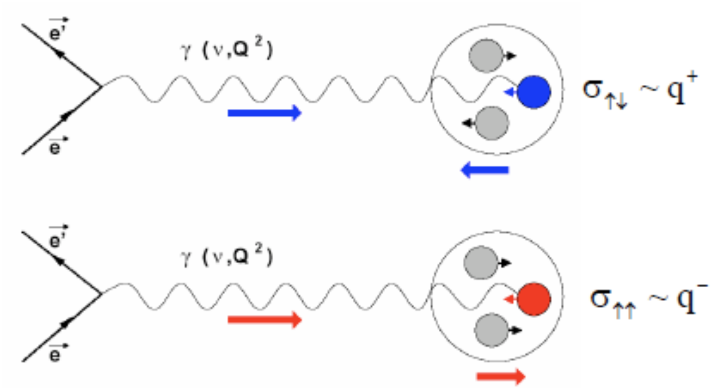


DIS and structure functions

$$\sigma \sim F_1(x) = \tfrac{1}{2}\sum_i e_q^2 q_i(x) \qquad F_2(x) \approx 2xF_1$$

$$\Delta\sigma = \overset{\leftrightarrows}{\sigma} - \overset{\rightrightarrows}{\sigma} \sim g_1(x) = \tfrac{1}{2}\sum_i e_q^2 \Delta q_i(x) \qquad g_2$$



$$\Delta q(x) = q(x)^{+} - q(x)^{-}$$

$$q(x) = q(x)^{+} + q(x)^{-}$$

+ **quark ↑↑ nucleon**

– **quark ↑↓ nucleon**

$$A_{meas} = \frac{1}{fP_TP_B}\left(\frac{N^{\leftrightarrows} - N^{\leftleftarrows}}{N^{\leftrightarrows} + N^{\leftleftarrows}}\right) \approx DA_1 \qquad A_1(x,Q^2) = \frac{\sigma_{\uparrow\downarrow} - \sigma_{\uparrow\uparrow}}{\sigma_{\uparrow\downarrow} + \sigma_{\uparrow\uparrow}} \approx \frac{\sum_q e_q^2\, \Delta q(x,Q^2)}{\sum_q e_q^2\, q(x,Q^2)} = \frac{g_1(x,Q^2)}{F_1(x,Q^2)}$$

DIS and structure functions

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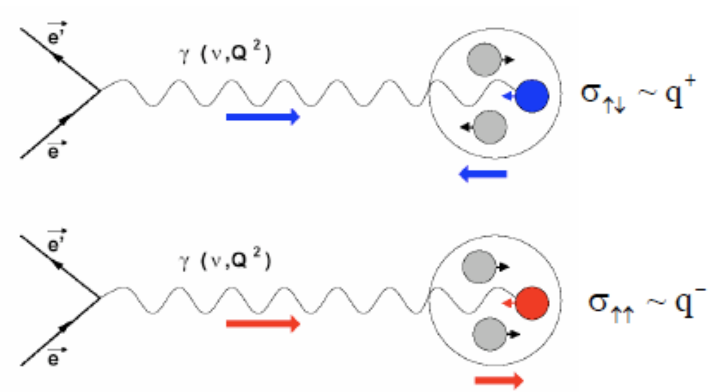

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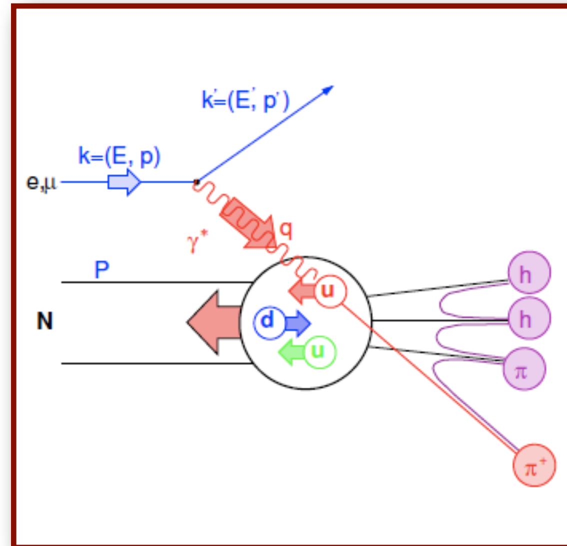

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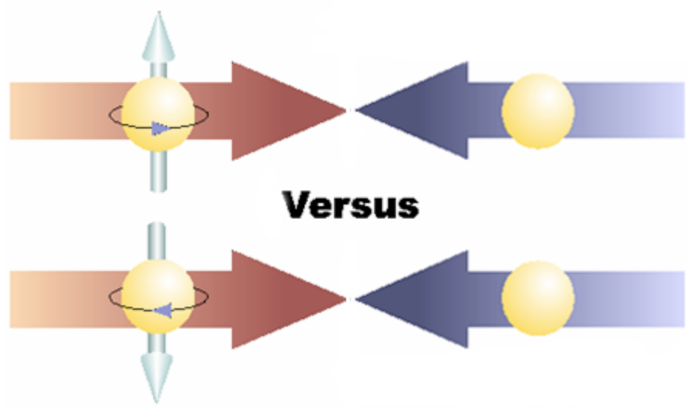
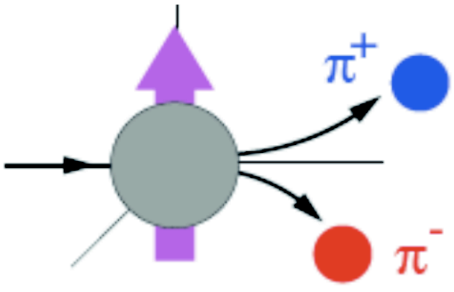
$$A_1^h(x,z,Q^2) = \frac{\sigma_{\uparrow\downarrow}^h - \sigma_{\uparrow\uparrow}^h}{\sigma_{\uparrow\downarrow}^h + \sigma_{\uparrow\uparrow}^h} \approx \frac{\sum_q e_q^2\,\Delta q(x,Q^2)D_q^h(z,Q^2)}{\sum_q e_q^2\, q(x,Q^2)D_q^h(z,Q^2)}$$

Not a first problem with spin and QCD

Single hadron-hadron asymmetry





QCD prediction:
A~ helicity flip amplitude ~m_q/p_T~ 0.001

$$A_N = \frac{\sigma^\uparrow - \sigma^\downarrow}{\sigma^\uparrow + \sigma^\downarrow}$$

Kane, Pumplin and Repko (1978)

Not a first problem with spin and QCD

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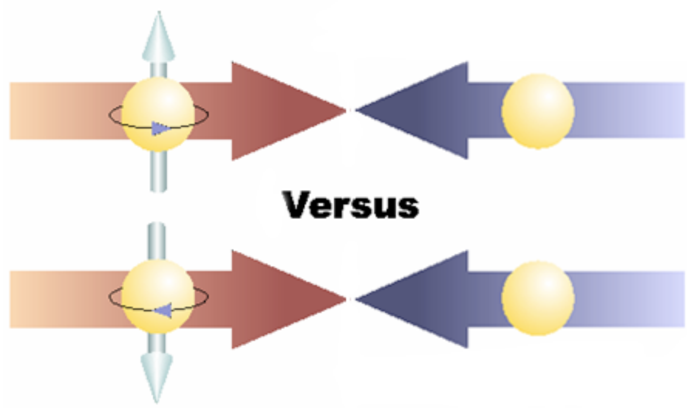
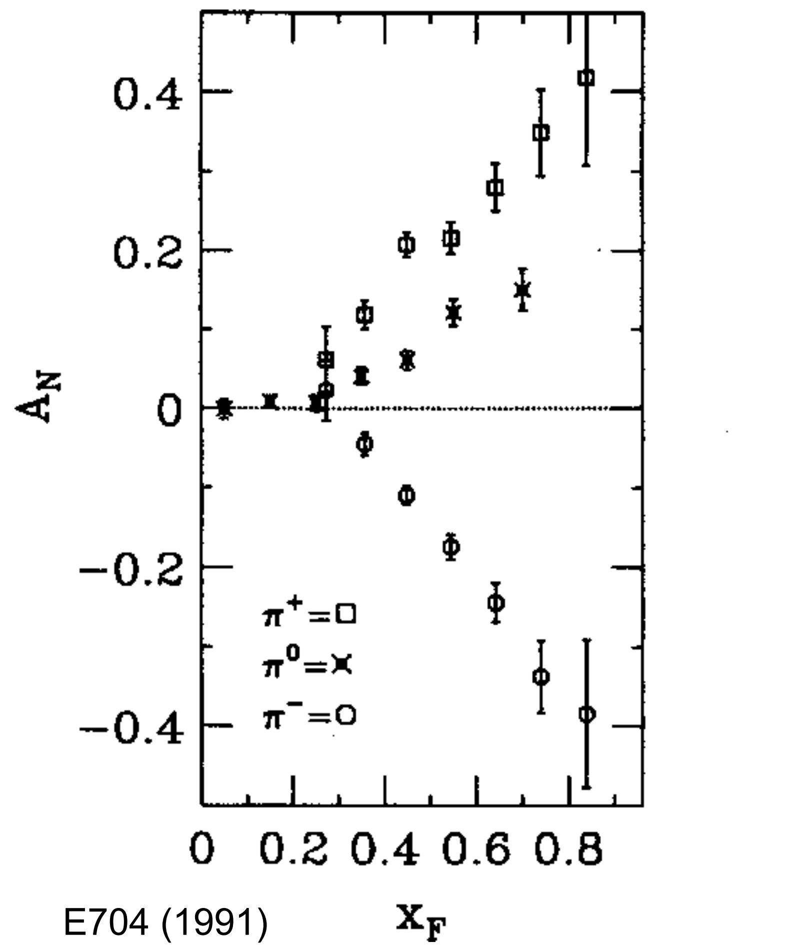




E704 (1991)

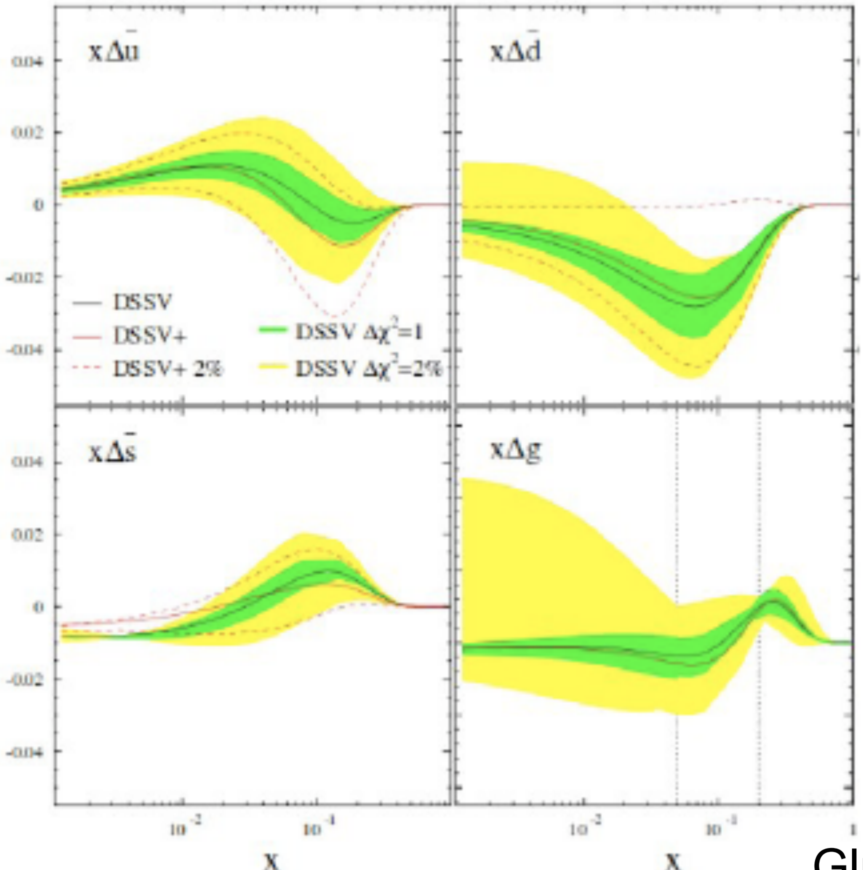
QCD prediction:
A~ helicity flip amplitude ~m_q/p_T~ 0.001

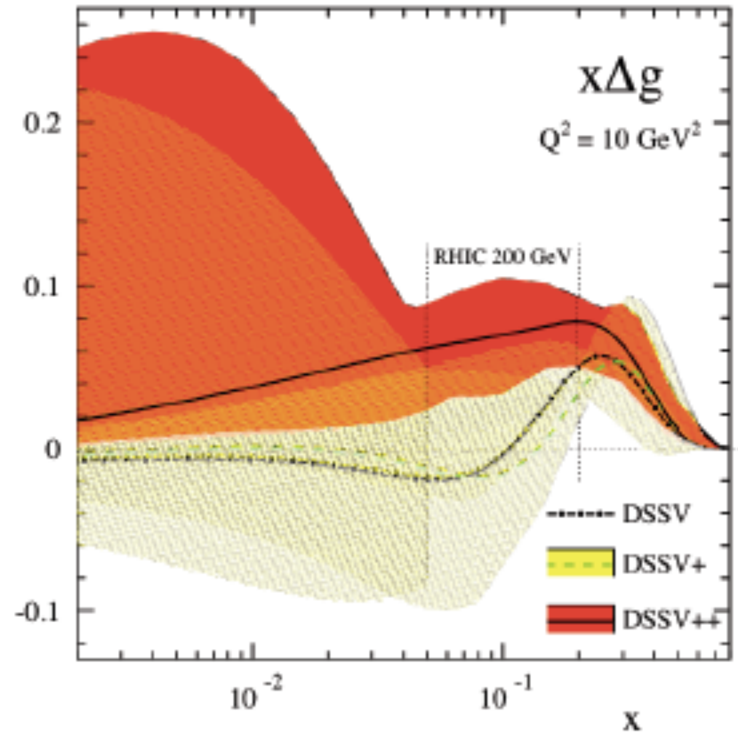
Kane, Pumplin and Repko (1978)

Confirmed (smaller effect) by RHIC:
STAR, BRAHMS and PHENIX,
COMPASS and HERMES
(27 GeV - 200 GeV)

NLO QCD fits - DSSV

Global fits (DSSV/DSSV+/DSSV++) include: spin-dependent DIS data, SIDIS data with identified π and K, and proton-proton data

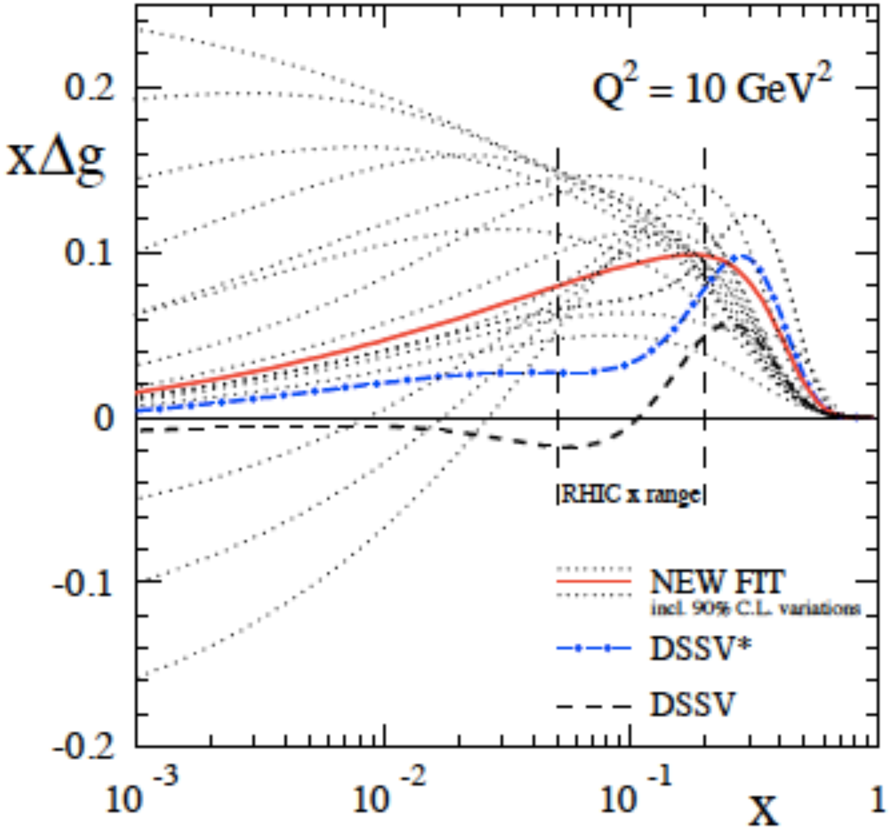


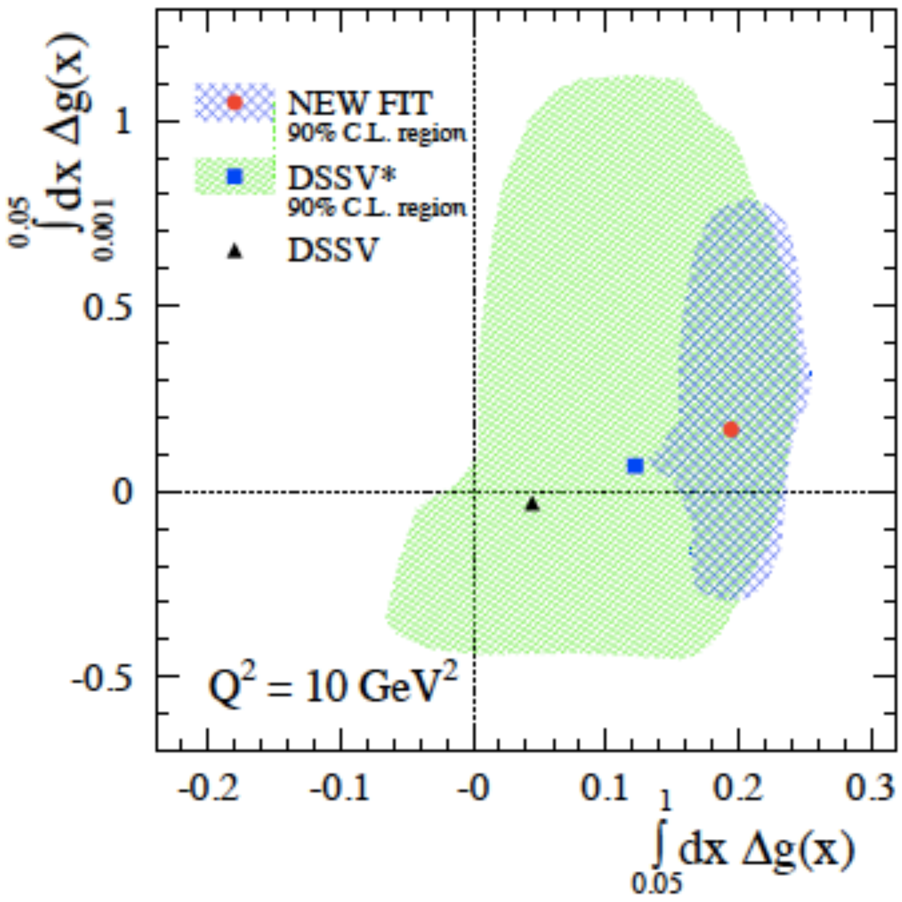


Latest PHENIX and STAR data included in DSSV++

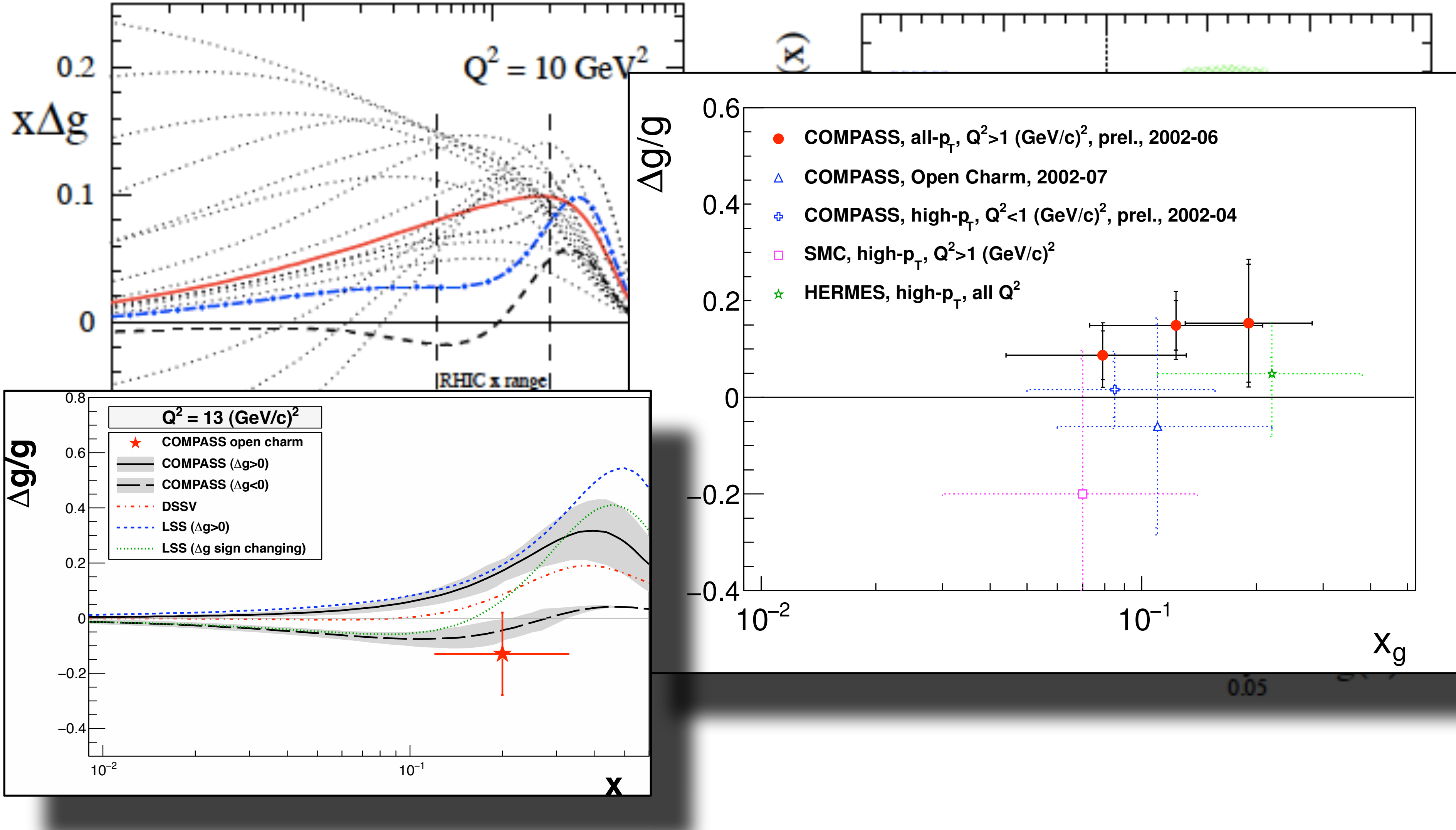
Gluons are not constrained by inclusive DIS data

Global QCD analysis & gluon polarisation



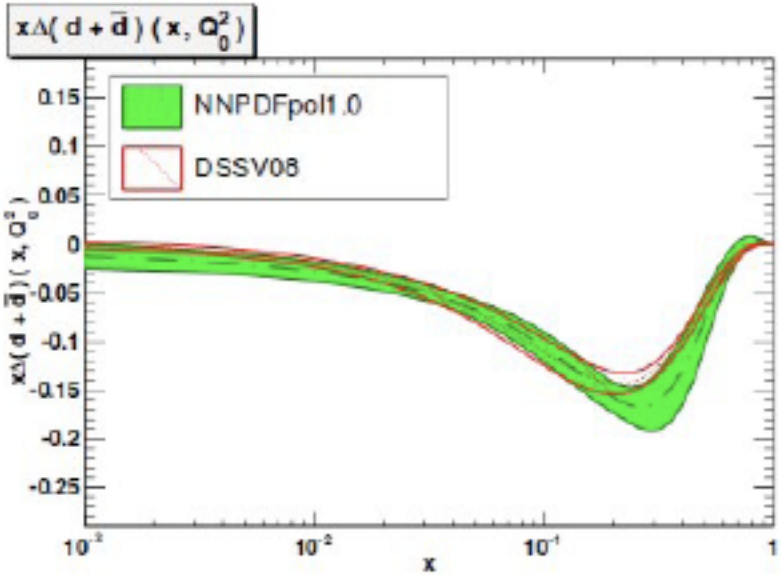


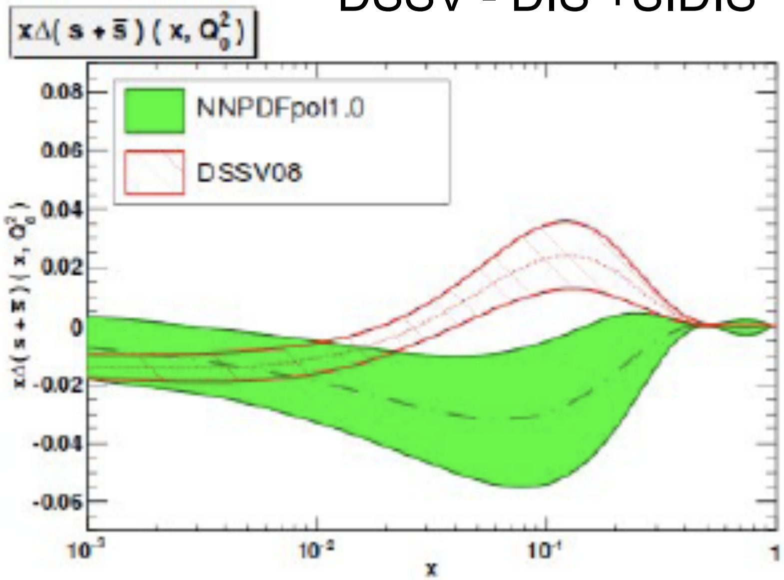
Global QCD analysis & gluon polarisation



Δs puzzle

NNPDF - only DIS
DSSV - DIS +SIDIS



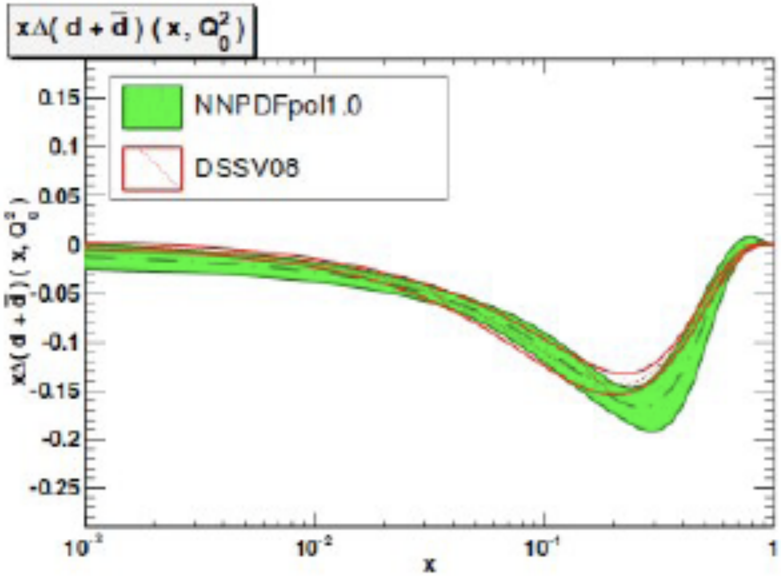


NNPDF, R.D.Ball et al. arXiv: 1303.7236

Δs puzzle

The lesson:
Important piece of information:
precise measurement of FFs

NNPDF - only DIS
DSSV - DIS +SIDIS



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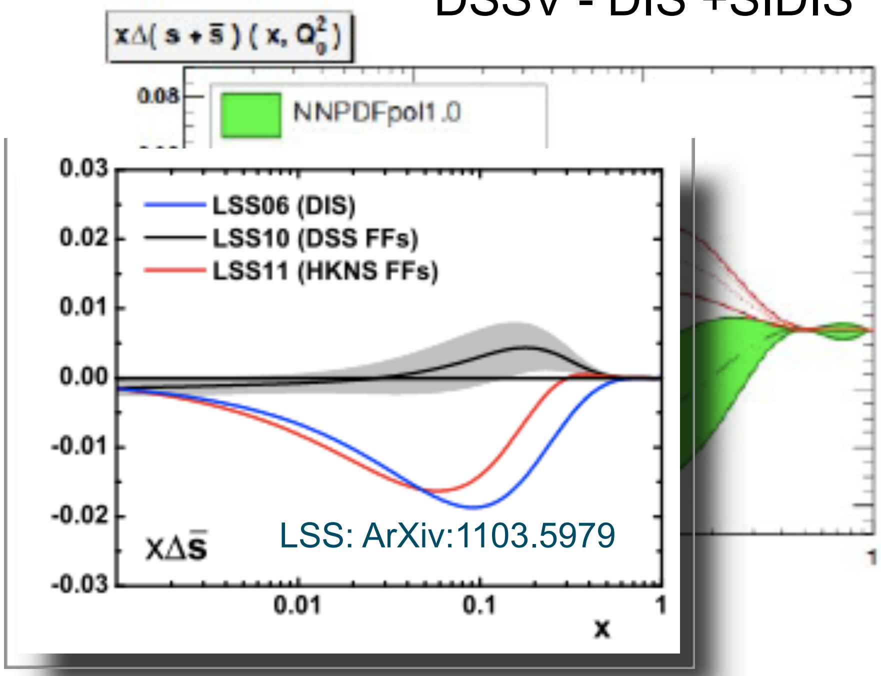
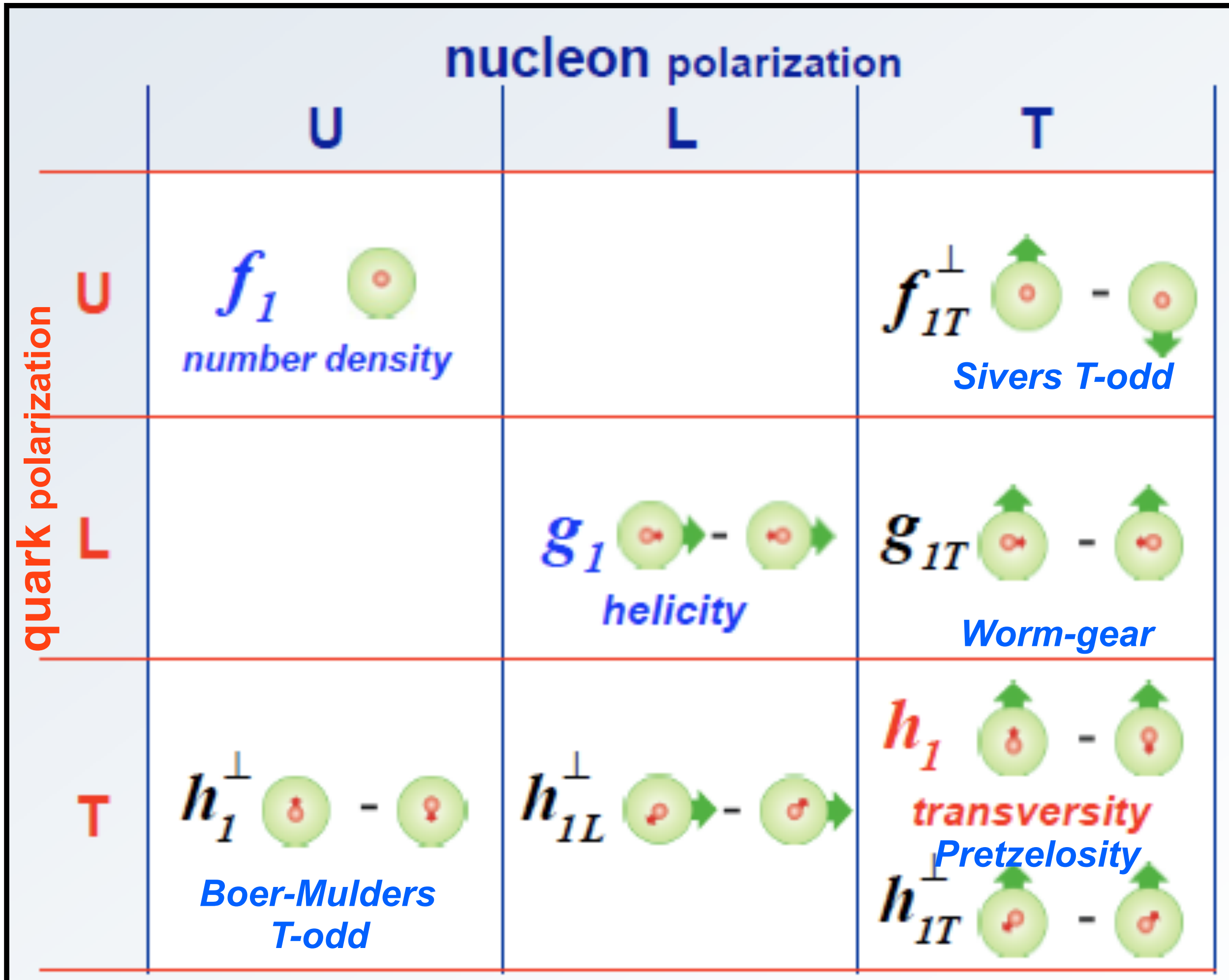


LSS: ArXiv:1103.5979

A catalog of PDFs
Transversely polarised target

Beyond collinear approximation - k_T dependence



LO, twist-2 - 8 independent functions to parameterize structure

Wigner function (1933)

Beyond collinear approximation - k_T dependence

$$W(x,p) = \int \Psi^*(x-\eta/2)\Psi(x+\eta/2)e^{ip\eta}d\eta,$$

$$\langle O(x,p)\rangle = \int O(x,p)W(x,p)dxdp.$$

Wigner function (1933)

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a gauge-link operator - key point here

A similar concept in QCD

$$\hat{W}(x,\vec{k}_\perp,\vec{S})_\eta = \int e^{ikz} < \vec{P},\vec{S}|\bar{\Psi}(0)W_\eta(0,z)\Psi(z)[\vec{P},\vec{S}| > |_{z^+=0}\frac{dz^- d^2 z_\perp}{(2\pi)^3}.$$

$$\begin{aligned}
\frac{1}{2}Tr(\gamma^+\hat{W}(x,\vec{k}_\perp,\vec{S}) &= f_1(x,\vec{k}_\perp) - \frac{\varepsilon^{jk}k_\perp^j S_T^k}{M}f_{1T}^\perp(x,\vec{k}_\perp),\\
\frac{1}{2}Tr(\gamma^+\gamma_5\hat{W}(x,\vec{k}_\perp,\vec{S}) &= S_L g_{1L}(x,\vec{k}_\perp) + \frac{\vec{k}_\perp\vec{S}_T}{M}g_{1T}(x,\vec{k}_\perp),\\
\frac{1}{2}Tr(i\sigma^{j+}\gamma_5\hat{W}(x,\vec{k}_\perp,\vec{S}) &= S_T^j h_1(x,\vec{k}_\perp) + S_L\frac{k_\perp^j}{M}h_{1L}^\perp(x,\vec{k}_\perp)\\
&+ \frac{(k_\perp^j k_\perp^k - \frac{1}{2}\vec{k}_\perp^2)S_T^k}{M^2}h_{1T}^\perp(x,\vec{k}_\perp) + \frac{\varepsilon^{jk}k_\perp^k}{M}h_1^\perp(x,\vec{k}_\perp).
\end{aligned}$$

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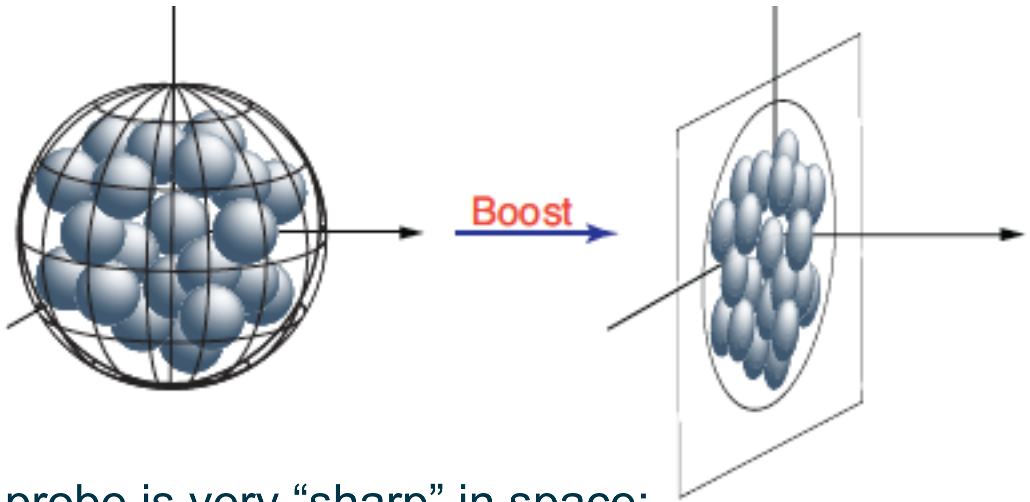
TMDs describes 3-Dimensional image of the nucleon in the momentum space. Some of them are related to Orbital Angular Momentum

$$\frac{1}{2}Tr(\gamma^+\gamma_5\hat{W}(x,\vec{k}_\perp,\vec{S})) = S_L g_{1L}(x,\vec{k}_\perp) + \frac{\vec{k}_\perp\cdot\vec{S}_T}{M} g_{1T}(x,\vec{k}_\perp),$$

$$\frac{1}{2}Tr(i\sigma^{j+}\gamma_5\hat{W}(x,\vec{k}_\perp,\vec{S})) = S_T^j h_1(x,\vec{k}_\perp) + S_L\frac{k_\perp^j}{M}h_{1L}^\perp(x,\vec{k}_\perp) + \frac{(k_\perp^j k_\perp^k - \frac{1}{2}\vec{k}_\perp^2)S_T^k}{M^2}h_{1T}^\perp(x,\vec{k}_\perp) + \frac{\varepsilon^{jk}k_\perp^k}{M}h_1^\perp(x,\vec{k}_\perp).$$

Transverse structure



Hard probe is very "sharp" in space:

Transverse size - $\frac{1}{Q} \ll 1\ \mathrm{fm}$

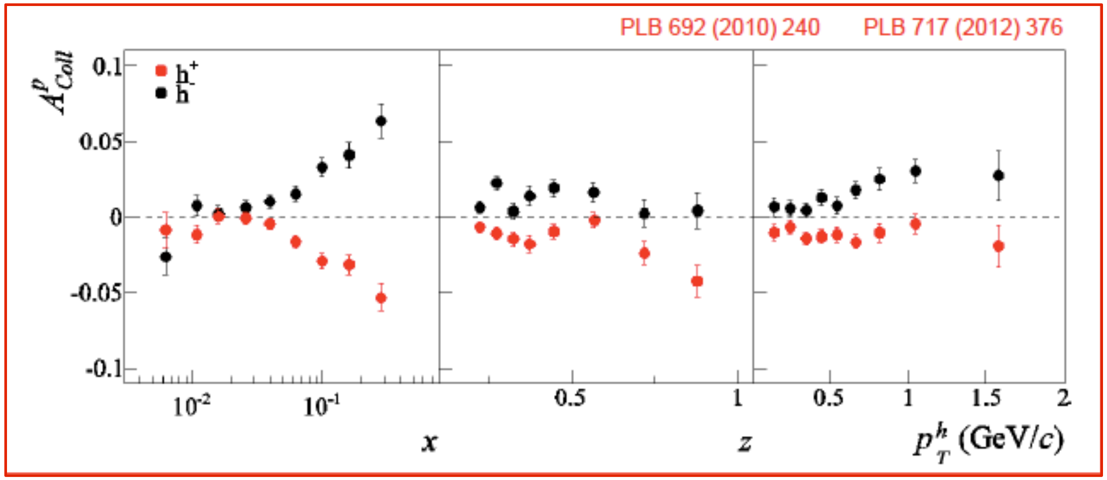
Longitudinal size ~ Lorentz contracted nucleon~ $\frac{1}{Q}$ no space!

QCD TMD factorization for cross sections with one large and one small momentum transfers

- Confined parton motion
- Quantum correlation between hadron property and parton motion,
- Parton properties influence hadronization

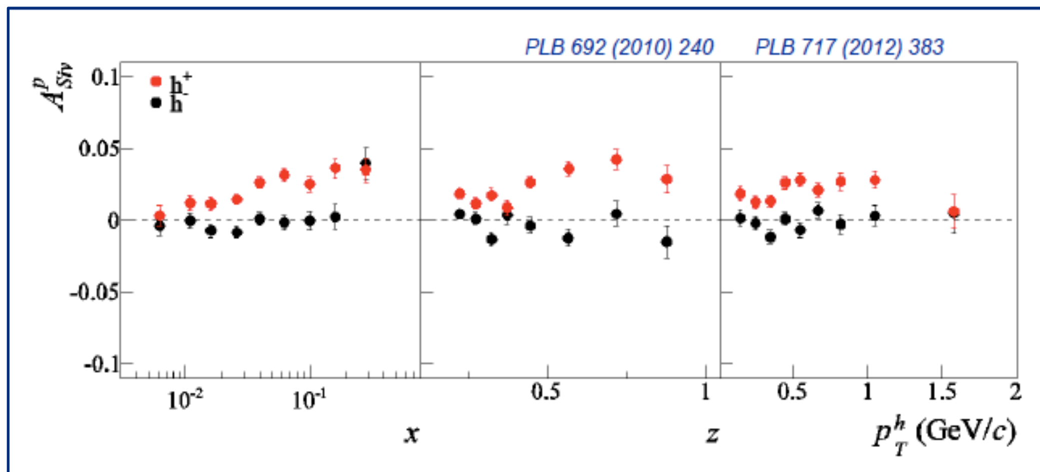
Transversity & Sivers function

Collins asymmetry, Collins FF confirmed non-zero by independent measurement at Belle



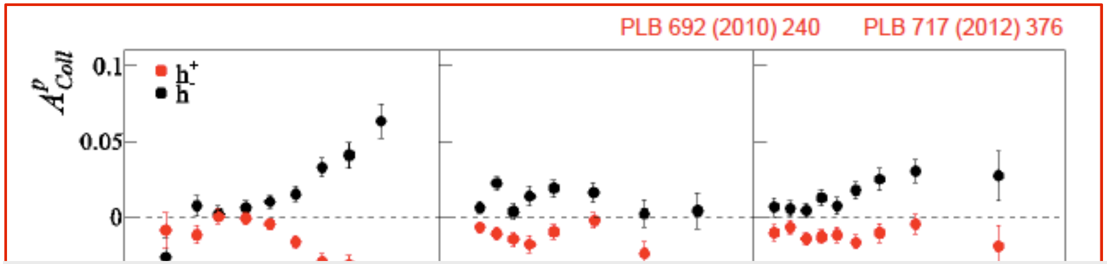
$$A_{Coll} \approx \frac{\sum_q e_q^2 \, h_1^q \otimes H_1^{\perp q}}{\sum_q e_q^2 \, f_1 \otimes D_1^q}$$

$$A_{Siv} \approx \frac{\sum_q e_q^2 \, f_{1T}^{\perp q} \otimes D_1^q}{\sum_q e_q^2 \, f_1 \otimes D_1^q}$$



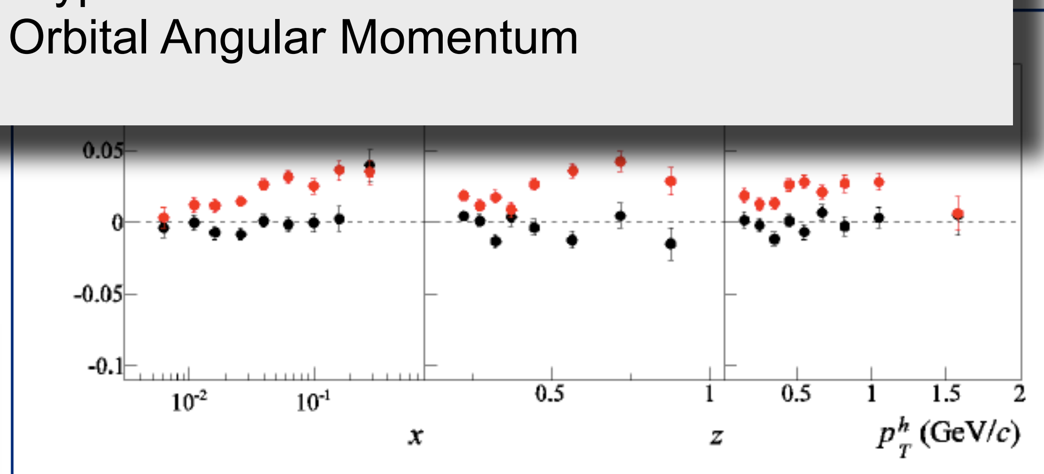
Transversity & Sivers function

Collins asymmetry, Collins FF confirmed non-zero by independent measurement at Belle



$$A_{Coll} \approx \frac{\sum_q e_q^2\; h_1^q \otimes H_1^{\perp q}}{\sum_q e_q^2\; f_1 \otimes D_1^q}$$

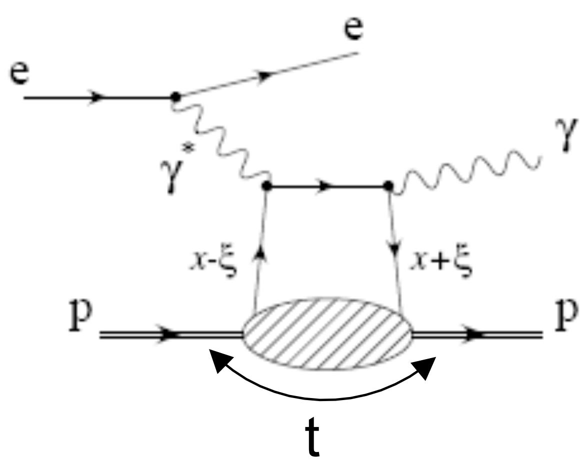
Non-zero effect (HERMES, COMPASS)
Correlation of tranverse quark motion and the nucleon spin
spin-orbit type correlation
requires non-zero Orbital Angular Momentum

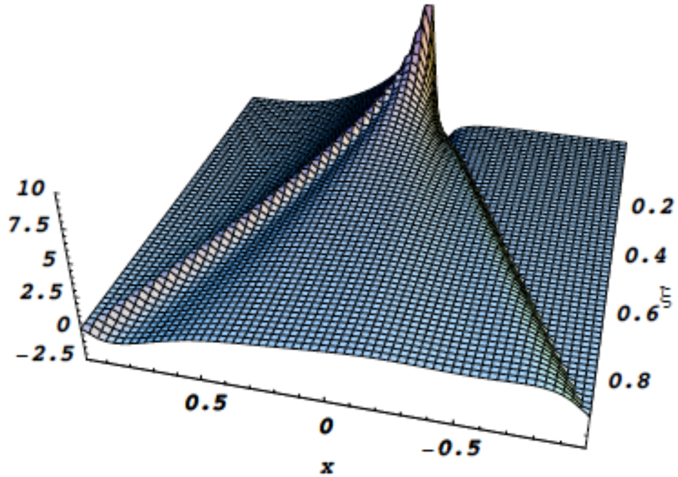


$$A_{Siv} \approx \frac{\sum_q e_q^2\; f_{1T}^{\perp q} \otimes D_1^q}{\sum_q e_q^2\; f_1 \otimes D_1^q}$$

GPDs and DVCS

$$H(x,\xi,t), \widetilde{H}, E, \widetilde{E}$$
$$H(x,0,0) = q(x)$$
$$\widetilde{H}(x,0,0) = \Delta q(x)$$
$$\int H(x,\xi,t)dx = F(t)$$

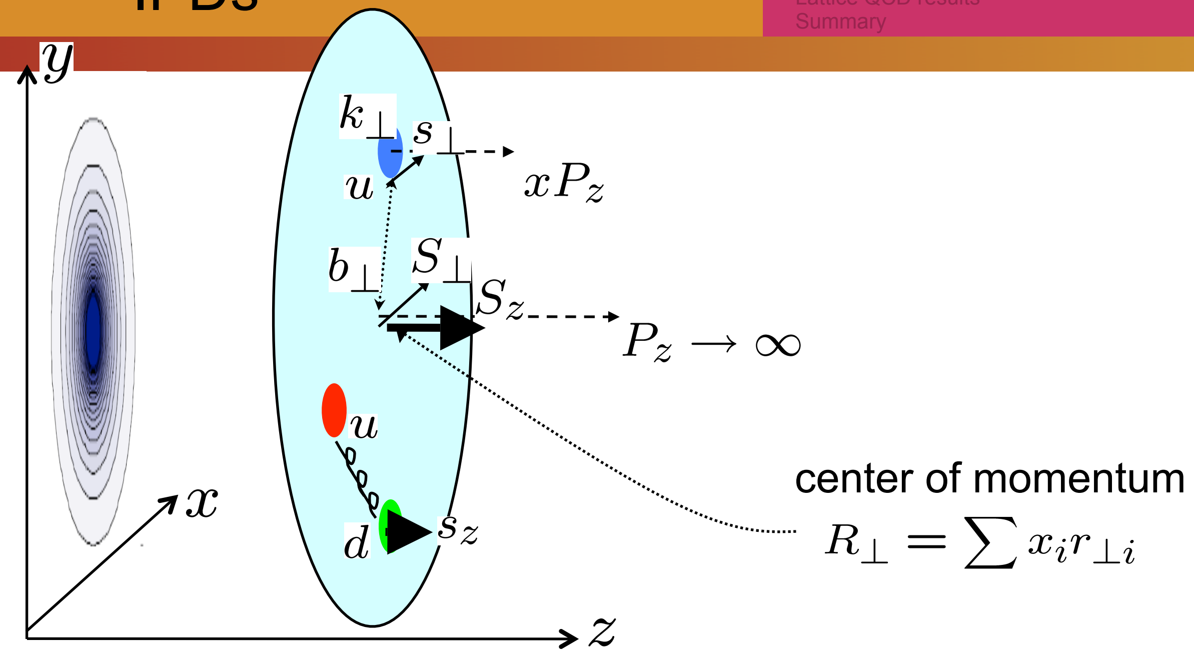




$$J^q(Q^2) = \frac{1}{2}\sum_{i=q,\bar{q}}\int_{-1}^{1} x(H^i(Q^2,x,\xi,0) + E^i(Q^2,x,\xi,0))dx$$
$$J^G(Q^2) = \frac{1}{2}\int_{-1}^{1} x(H^G(Q^2,x,\xi,0) + E^G(Q^2,x,\xi,0))dx$$

Ji’s sum rule

GPDs and Impact Parameter PDFs
IPDs



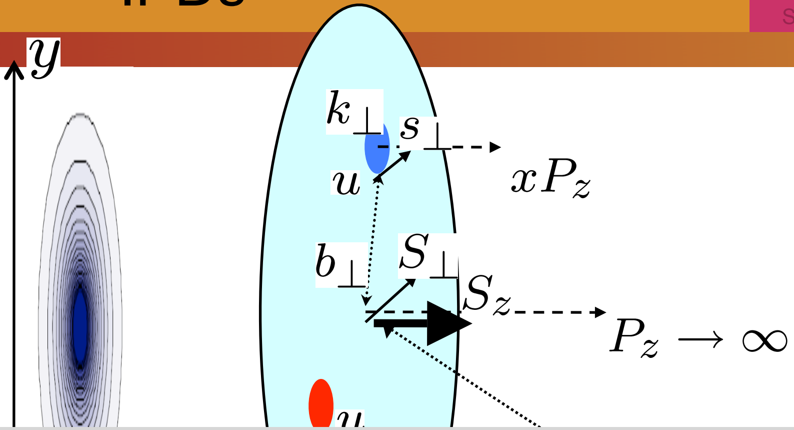
$$\mathcal{H}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} H(x,0,-\vec{\Delta}_\perp^2).$$

$$\tilde{\mathcal{H}}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} \tilde{H}(x,0,-\vec{\Delta}_\perp^2),$$

$$\mathcal{E}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} E(x,0,-\vec{\Delta}_\perp^2).$$

3-Dimensional image of nucleon in the mixed transverse plane-longitudinal momentum space

GPDs and Impact Parameter PDFs
IPDs



Complementary to TMDs - another “tool ” for 3D imaging
Burhardt PRD (2000) - GPDs related to TMDs via FT?
Not working! (only 1-gluon exchange app.)

$$\mathcal{H}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} H(x,0,-\vec{\Delta}_\perp^2).$$

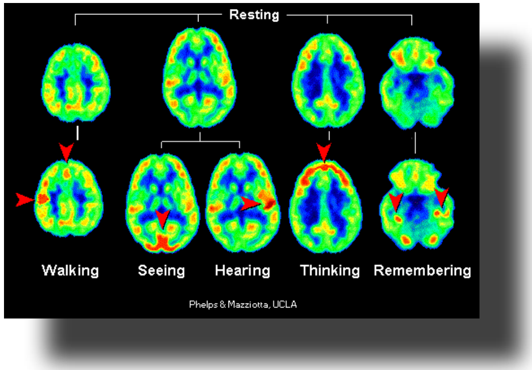
$$\tilde{\mathcal{H}}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} \tilde{H}(x,0,-\vec{\Delta}_\perp^2),$$

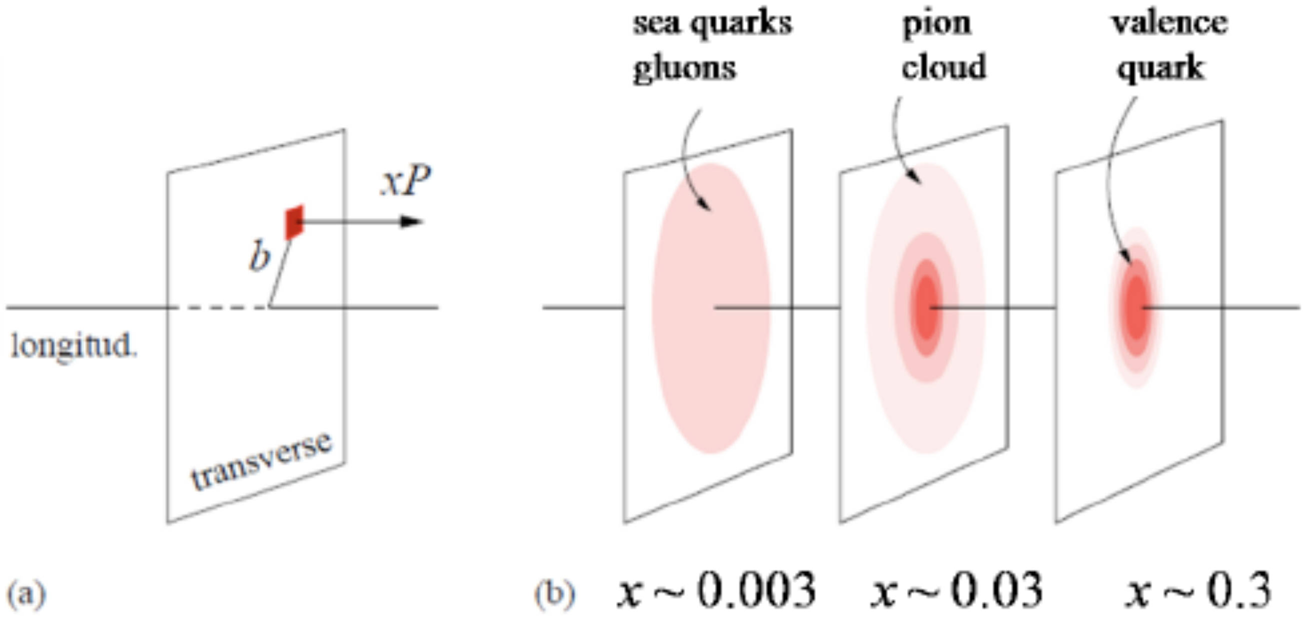
$$\mathcal{E}(x,\vec{b}_\perp) = \int \frac{d^2\Delta_\perp}{2\pi} e^{i\vec{\Delta}_\perp.\vec{b}_\perp} E(x,0,-\vec{\Delta}_\perp^2).$$

3-Dimensional image of nucleon in the mixed transverse plane-longitudinal momentum space

nucleon “tomography”

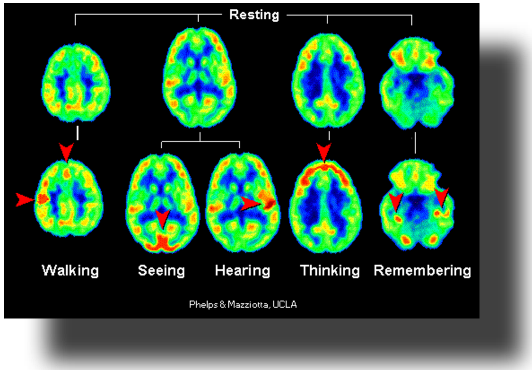
slices in longitudinal momentum x

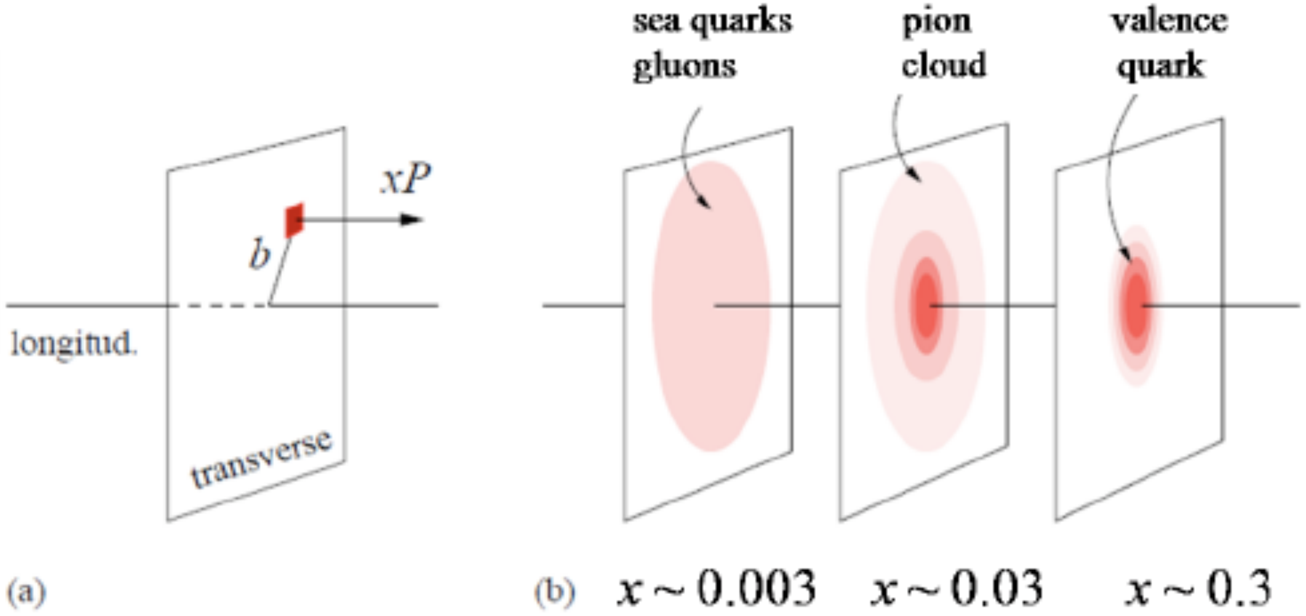




nucleon “tomography”

slices in longitudinal momentum x

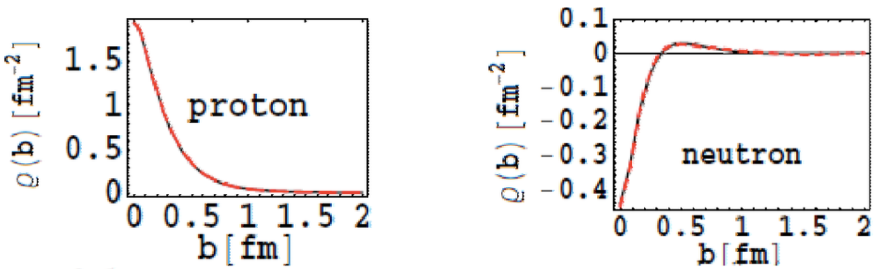




The “big” question: How color is distributed inside the hadron?

charge - EM formactors

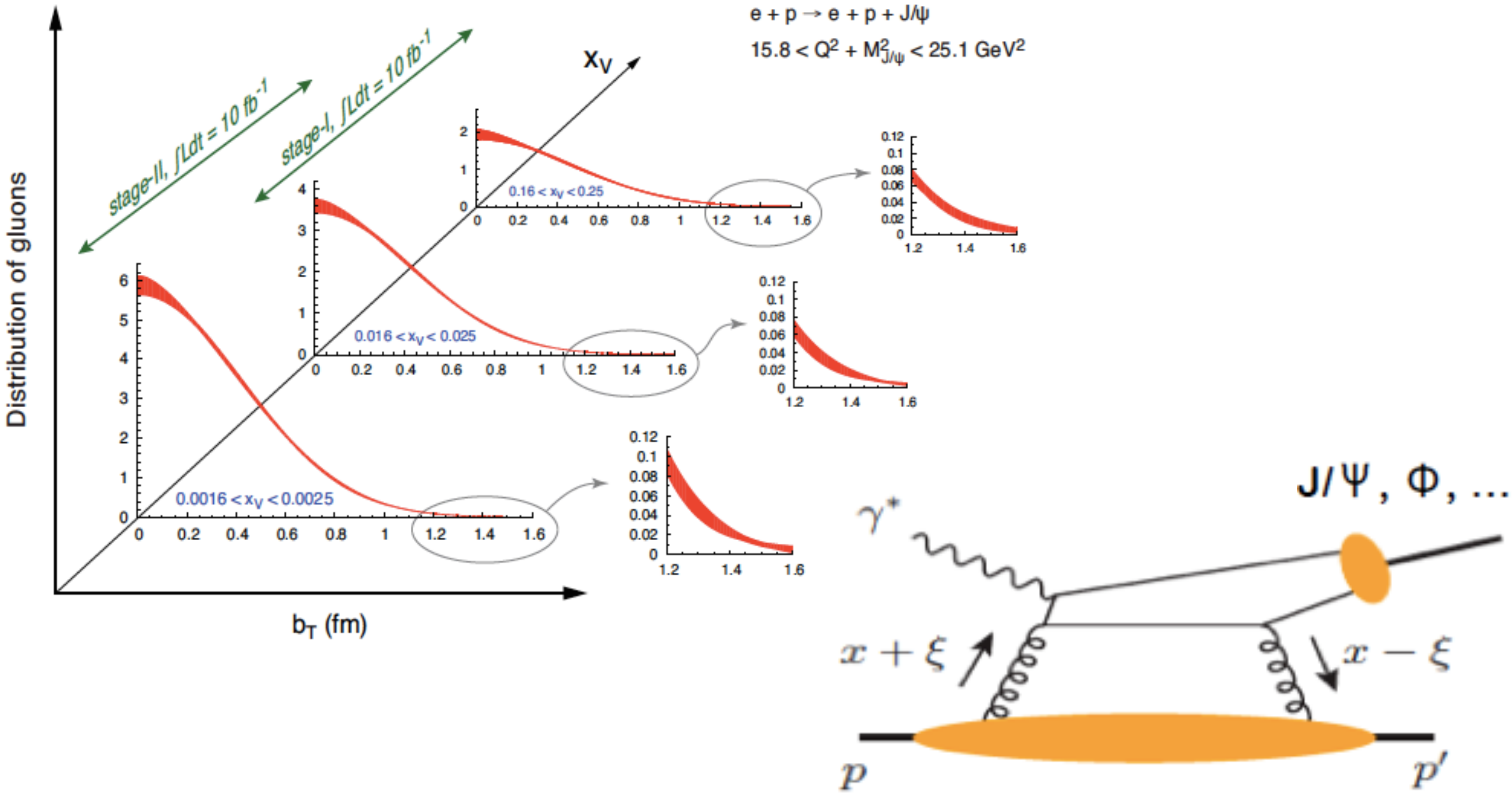
Charge distributions



color - hadron is colorless!

need exchange a localized, colorless object

Images of gluons from exclusive J/Ψ production - simulations for EIC



Generalized Wigner function

Wigner Distributions
Parton distributions in the Phase Space

FT $\vec{b}_\perp \leftrightarrow \vec{\Delta}_\perp$

Generalized Transverse Momentum Dependent Parton Distributions (GTMDs)

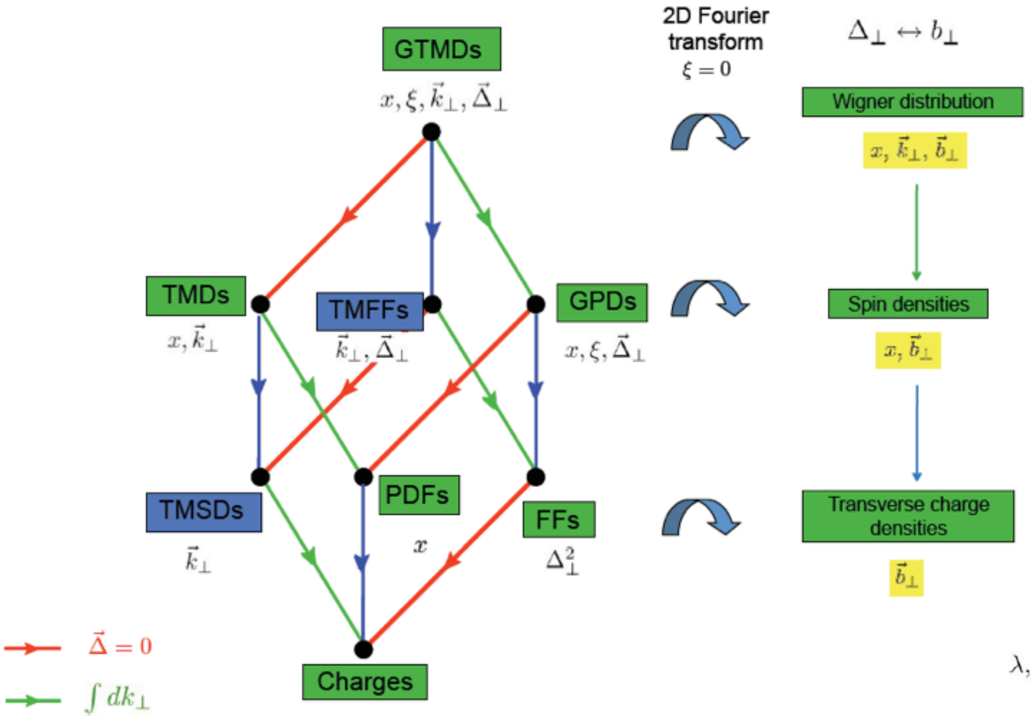
GPDs

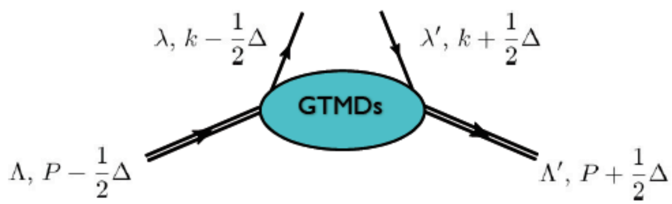
TMDs

spin and orbital angular momentum structure of the nucleon

insights from quark model calculations

Generalized Wigner function





Quark polarization

$$\boxed{W^{[\Gamma]}_{\Lambda'\Lambda} = \frac{1}{2}\int \frac{\mathrm{d}z^- \mathrm{d}^2 z_\perp}{(2\pi)^3} \langle P', \Lambda' | \bar{\psi}^q_{\lambda'}(-\frac{z}{2})\, \Gamma\, \mathcal{W}(-\frac{z}{2}, \frac{z}{2}|n) \psi^q_\lambda(\frac{z}{2}) | P, \Lambda \rangle e^{i(xP^+ z^- - \vec{k}_\perp \cdot \vec{z}_\perp)}}$$

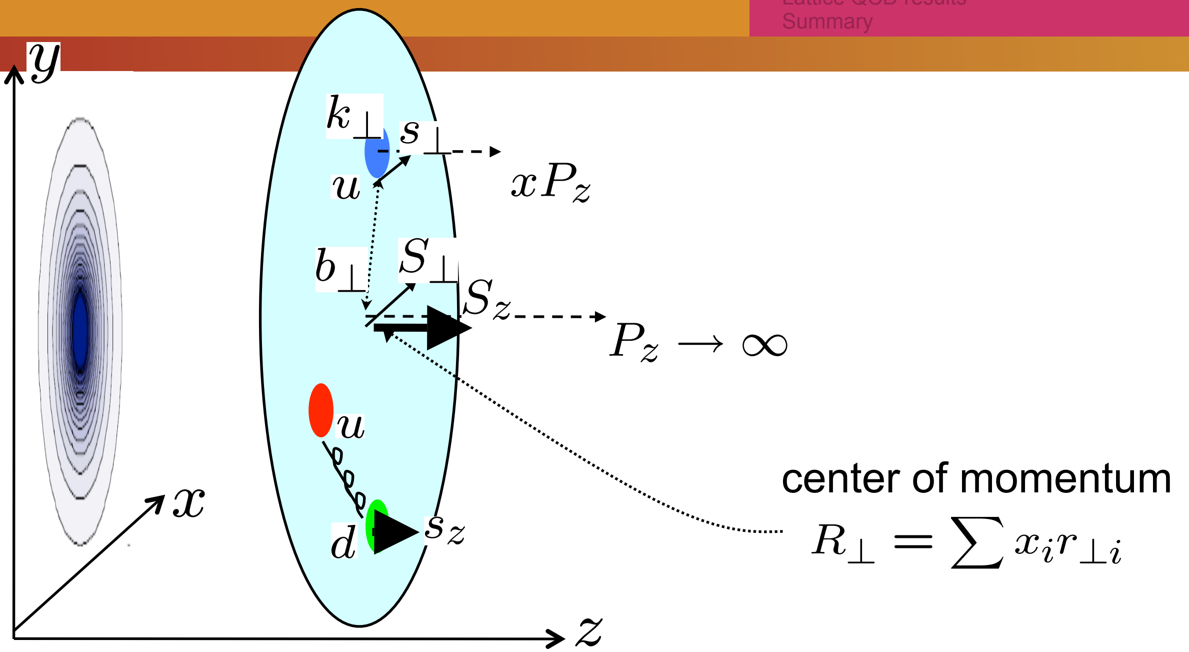
Nucleon polarization

4 X 4 =16 polarizations ⟷ 16 complex GTMDs (at twist-2)

$W^{\Gamma}_{\Lambda',\Lambda}(x, \xi, \vec{k}_\perp, \vec{\Delta}_\perp)$

Transverse distortion

Burhardt,2000

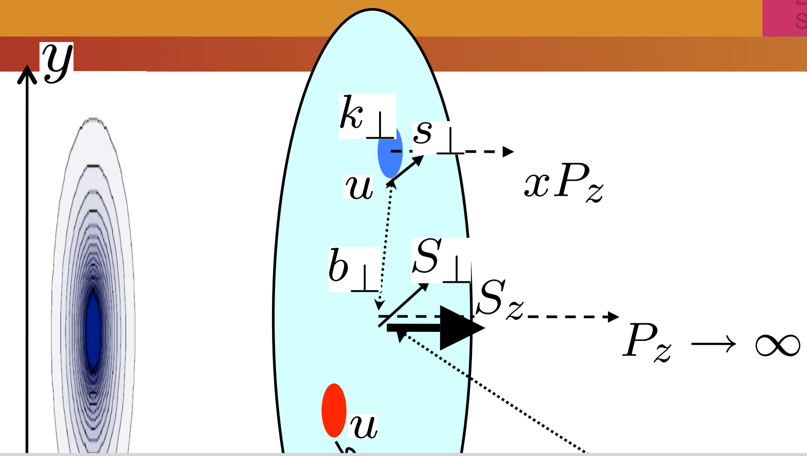


For a transversely polarized nucleon (e.g. polarized in the $+\hat{x}$-direction) the IPD $q_{\hat{x}}(x, \vec{b}_\perp)$ is no longer symmetric due to the non-zero value of the spin-flip GPD E. This deformation is described by the gradient of the Fourier transform of E:

$$q_{\hat{x}}(x, \vec{b}_\perp) = \mathcal{H}(x, \vec{b}_\perp) - \boxed{\frac{1}{2M} \frac{\partial}{\partial b_y} \mathcal{E}(x, \vec{b}_\perp).}$$

Transverse distortion

Burhardt,2000



Non-zero spin-flip GPD E function means non-zero Orbital Angular Momentum !

For a transversely polarized nucleon (e.g. polarized in the $+\hat{x}$-direction) the IPD $q_{\hat{x}}(x, \vec{b}_\perp)$ is no longer symmetric due to the non-zero value of the spin-flip GPD E. This deformation is described by the gradient of the Fourier transform of E:

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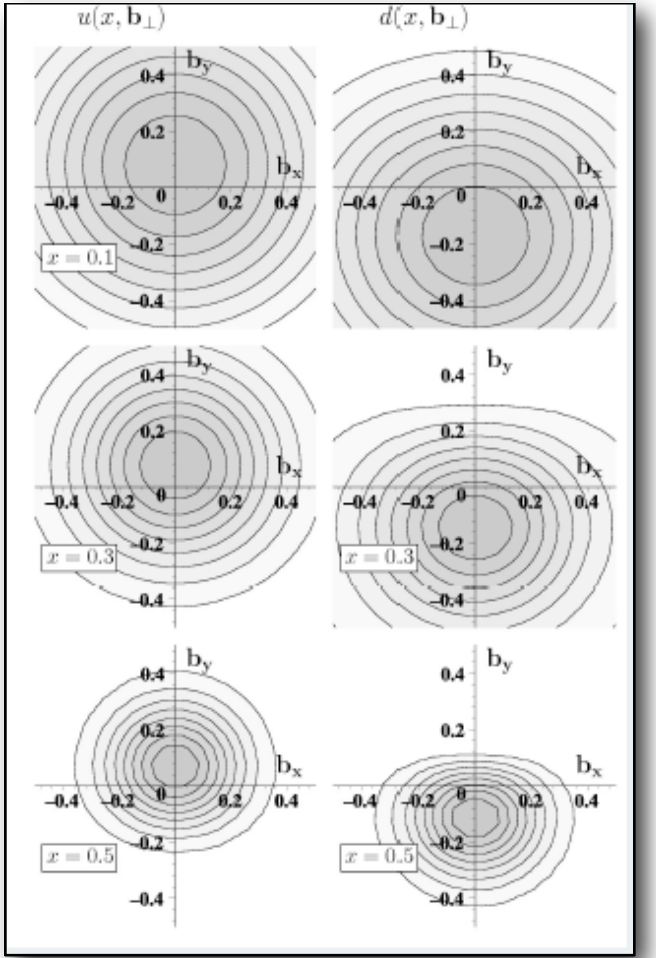
Sivers function and GPD



$$\langle b_y^q \rangle \equiv \int dx \int d^2 b_\perp q(x, \mathbf{b}_\perp) b_y = \frac{1}{2M} \int dx E_q(x,0,0) = \frac{\kappa_q}{2M}$$

Deformation details are model-dependent but the size and directions is determined by anomalous magnetic moments of proton and neutron.

$$\kappa^p = 1.913 = \frac{2}{3}\kappa_u^p - \frac{1}{3}\kappa_d^p + \ldots$$

- u-quarks: $\kappa_u^p = 2\kappa_p + \kappa_n = 1.673$

$\hookrightarrow$ shift in $+\hat{y}$ direction

- d-quarks: $\kappa_d^p = 2\kappa_n + \kappa_p = -2.033$

$\hookrightarrow$ shift in $-\hat{y}$ direction

- $\langle b_y^q \rangle = \mathcal{O}(\pm 0.2 fm)$!!!!

Deformation condirmed by LQCD!

Angular Momentum

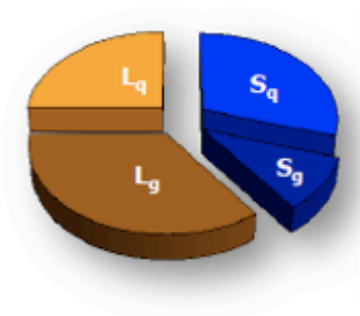
Orbital momentum

Orbital angular momentum

Observable ***Quasi*-observable** **Not observable**

Canonical

[Jaffe-Manohar (1990)]



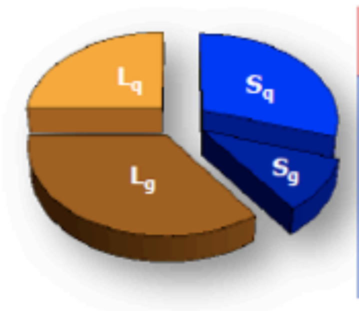
$$\vec{S}_q = \frac{1}{2}\int d^3r\, \psi^\dagger \vec{\Sigma}\psi$$

$$\vec{L}_q = \int d^3r\, \psi^\dagger \vec{r} \times (-i\vec{\nabla})\psi$$

$$\vec{S}_g = \int d^3r\, \vec{E}^a \times \vec{A}^a$$

$$\vec{L}_g = \int d^3r\, E^{ai}\vec{r} \times \vec{\nabla} A^{ai}$$

[Chen *et al.* (2008)]



$$\vec{S}_q = \frac{1}{2}\int d^3r\, \psi^\dagger \vec{\Sigma}\psi$$

$$\vec{L}_q = \int d^3r\, \psi^\dagger \vec{r} \times (-i\vec{D}_{\text{pure}})\psi$$

$$\vec{S}_g = \int d^3r\, \vec{E}^a \times \vec{A}^a_{\text{phys}}$$

$$\vec{L}_g = \int d^3r\, E^{ai}\vec{r} \times \vec{\mathcal{D}}_{\text{pure}} A^{ai}_{\text{phys}}$$

Kinetic

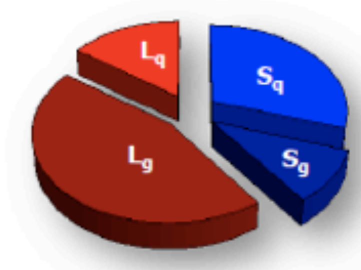
[Ji (1997)]

$$\vec{S}_q = \frac{1}{2}\int d^3r\, \psi^\dagger \vec{\Sigma}\psi$$

$$\vec{L}_q = \int d^3r\, \psi^\dagger \vec{r} \times (-i\vec{D})\psi$$

$$\vec{J}_g = \int d^3r\, \vec{r} \times (\vec{E}^a \times \vec{B}^a)$$

[Wakamatsu (2010)]



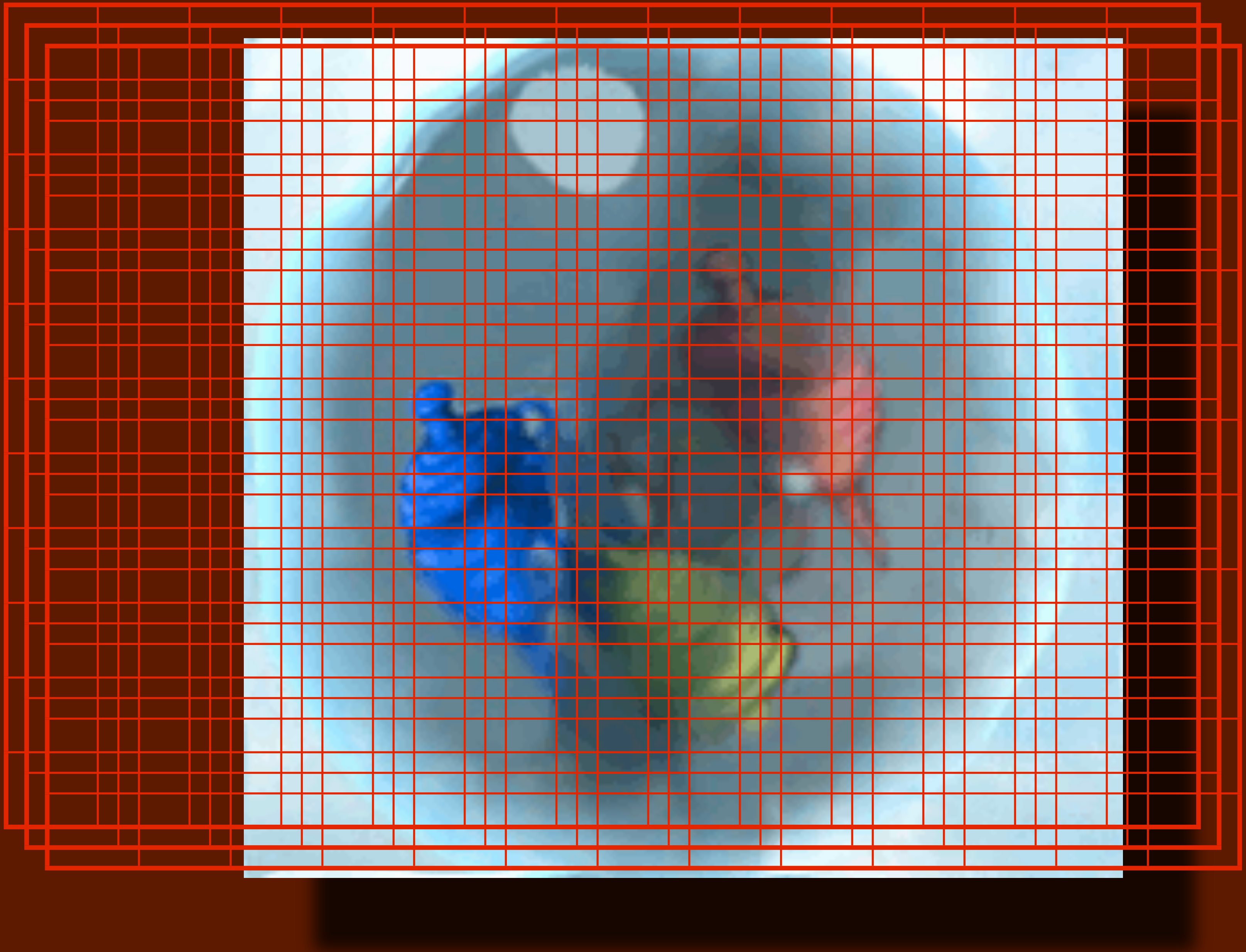
$$\vec{S}_q = \frac{1}{2}\int d^3r\, \psi^\dagger \vec{\Sigma}\psi$$

$$\vec{L}_q = \int d^3r\, \psi^\dagger \vec{r} \times (-i\vec{D})\psi$$

$$\vec{S}_g = \int d^3r\, \vec{E}^a \times \vec{A}^a_{\text{phys}}$$

$$\vec{L}_g = \int d^3r\, \vec{r} \times (\vec{E}^a \times \vec{B}^a) - \int d^3r\, \vec{E}^a \times \vec{A}^a_{\text{phys}}$$

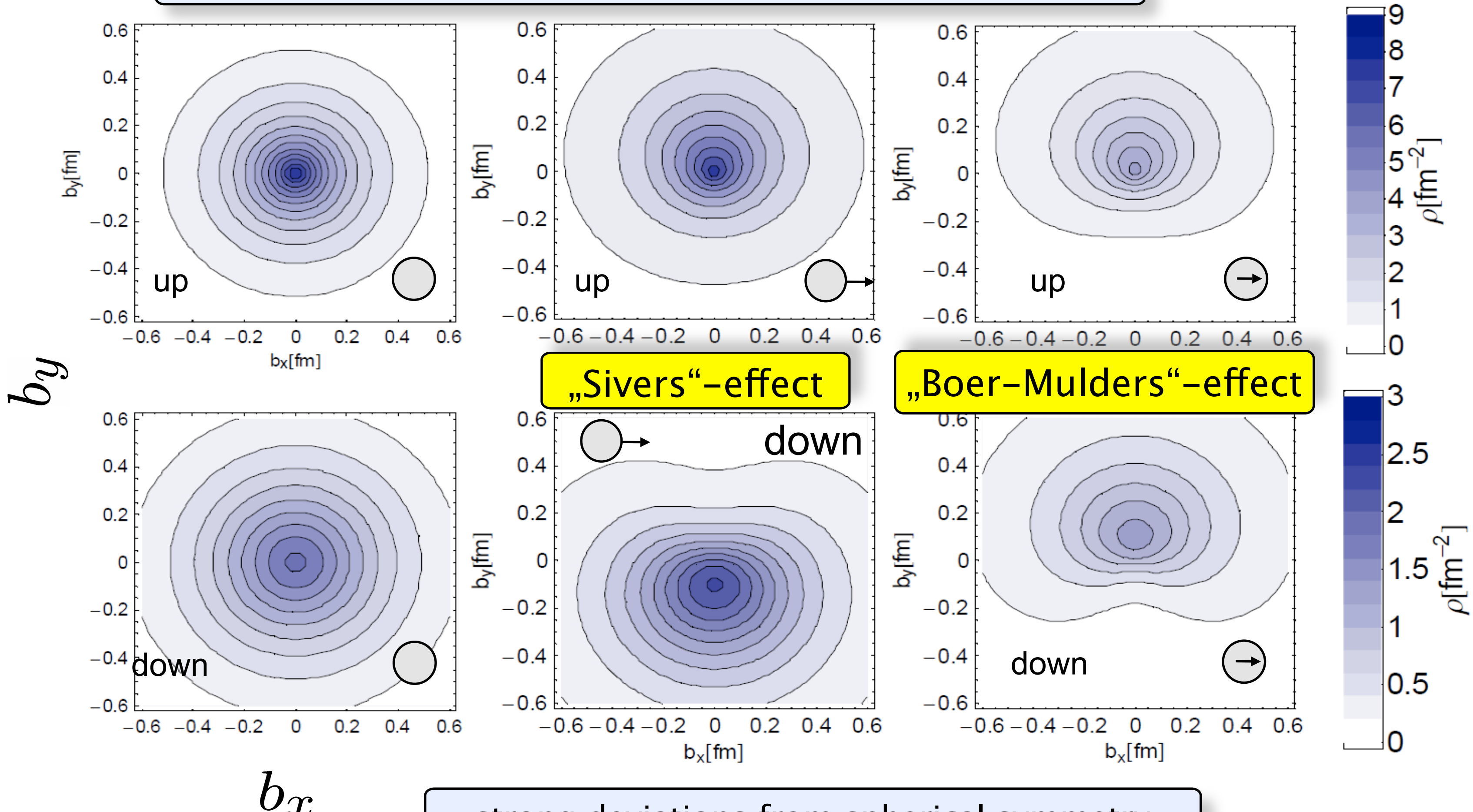
Lattice QCD results



Transverse quark distribution deformation from lattice QCD

Lowest x-moments of quark densities in coordinate space

QCDSF n_f=2 Clover, Phys.Rev.Lett. 2007 [hep-lat/0612032]



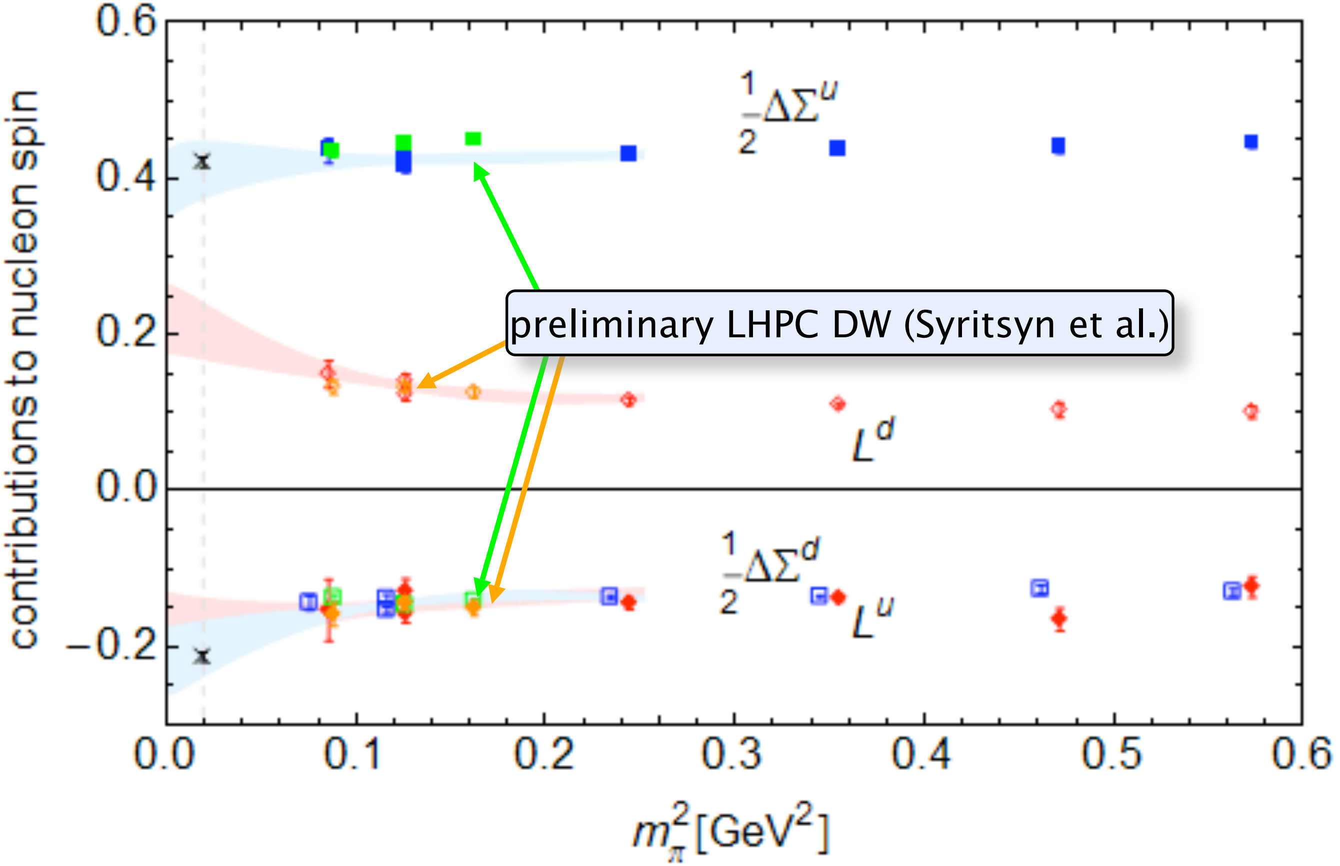
strong deviations from spherical symmetry

Spin structure @ lattice QCD

P. Haegler

$$L_q = J_q - \Delta\Sigma_q/2$$

LHPC n_f=2+1 mixed
arXiv:1001.3620



$\overline{\mathrm{MS}}$ at 4 GeV^2

$J^u \approx 0.236 \pm 0.006 \,\widehat{=}\, 48\%$ of $1/2$

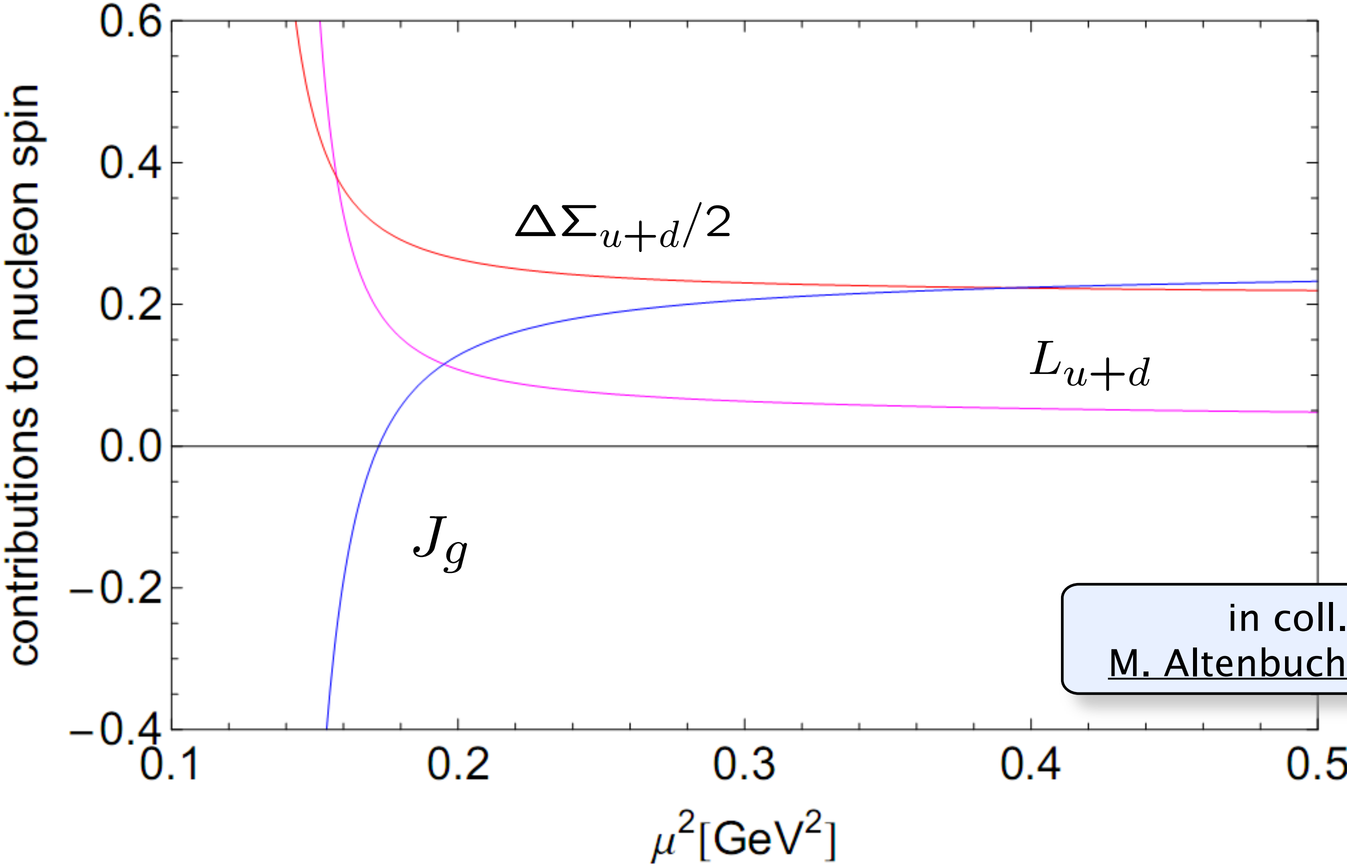
$J^d \approx 0.002 \pm 0.004$

$L^d \approx -L^u \approx 0.185 \pm 0.06 \approx 36\%$ of $1/2$

$L^{u+d} \approx 0.030 \pm 0.012 \approx 6\%$ of $1/2$

Lattice vs relativistic quark model

(Wakamatsu 2005; Thomas, PRL 2008)



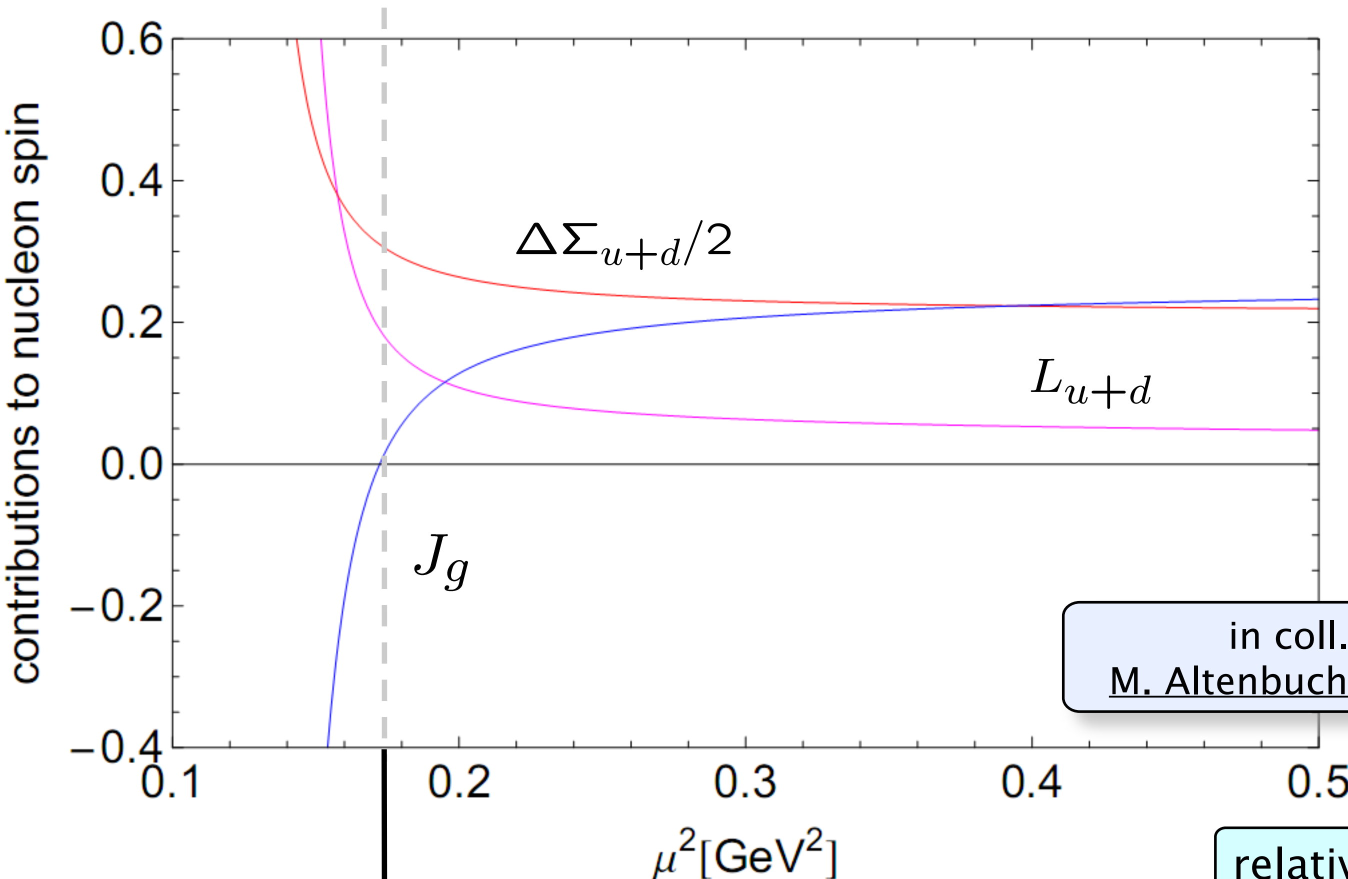
$\leftarrow$ 4 GeV^2

$\leftarrow$ 4 GeV^2

in coll. with LHPC and
M. Altenbuchinger, W. Weise (TUM)

Lattice vs relativistic quark model

(Wakamatsu 2005; Thomas, PRL 2008)



$\leftarrow$ 4 GeV^2

$\leftarrow$ 4 GeV^2

in coll. with LHPC and
M. Altenbuchinger, W. Weise (TUM)

relativistic quark model

$L^{u+d} \approx 0.19 \approx 38\%$ of $1/2$
$\Delta\Sigma^{u+d}/2 \approx 0.31 \approx 62\%$ of $1/2$

lattice + evolution

$L^{u+d} \approx 0.18 \approx 36\%$ of $1/2$
$\Delta\Sigma^{u+d}/2 \approx 0.32 \approx 64\%$ of $1/2$

Lattice vs relativistic quark model

(Wakamatsu 2005; Thomas, PRL 2008)

But:

1. Observed ΔΣ is lower than 0.65. Adding pion cloud + 1G exchange to the relativistic chiral model it is possible to get 0.35!
2. The LQCD again disagrees with “improved” chiral models.
3. On the other hand: starting value for backward QCD evolution from LQCD is higher than measured - still a lot of work to do

First link between chiral effective QCD models and perturbative QCD seen via LQCD

0.6

−0.4

0.1 0.2 0.3 0.4 0.5

$\mu^2[\mathrm{GeV}^2]$

$L^{u+d} \approx 0.19 \approx 38\%$ of $1/2$

$\Delta\Sigma^{u+d}/2 \approx 0.31 \approx 62\%$ of $1/2$

lattice + evolution

relativistic quark model

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Summary

Fast moving nucleon is a three-dimensional object

- Orbital angular momentum of valence quarks have opposite sign and compensate ~0 for higher scale; non-zero for small scale limit (Lattice QCD)
- If gluon polarisation is small (new data - maybe not) then nucleon spin seems to be composed of spin of quarks and orbital angular momentum of gluons.
- New measurements are needed: gluon polarisation (higher precision), GPDs, TMDs and Drell-Yann
- New probe for nucleon structure - W and Z bozons (RHIC, not cover in this talk)

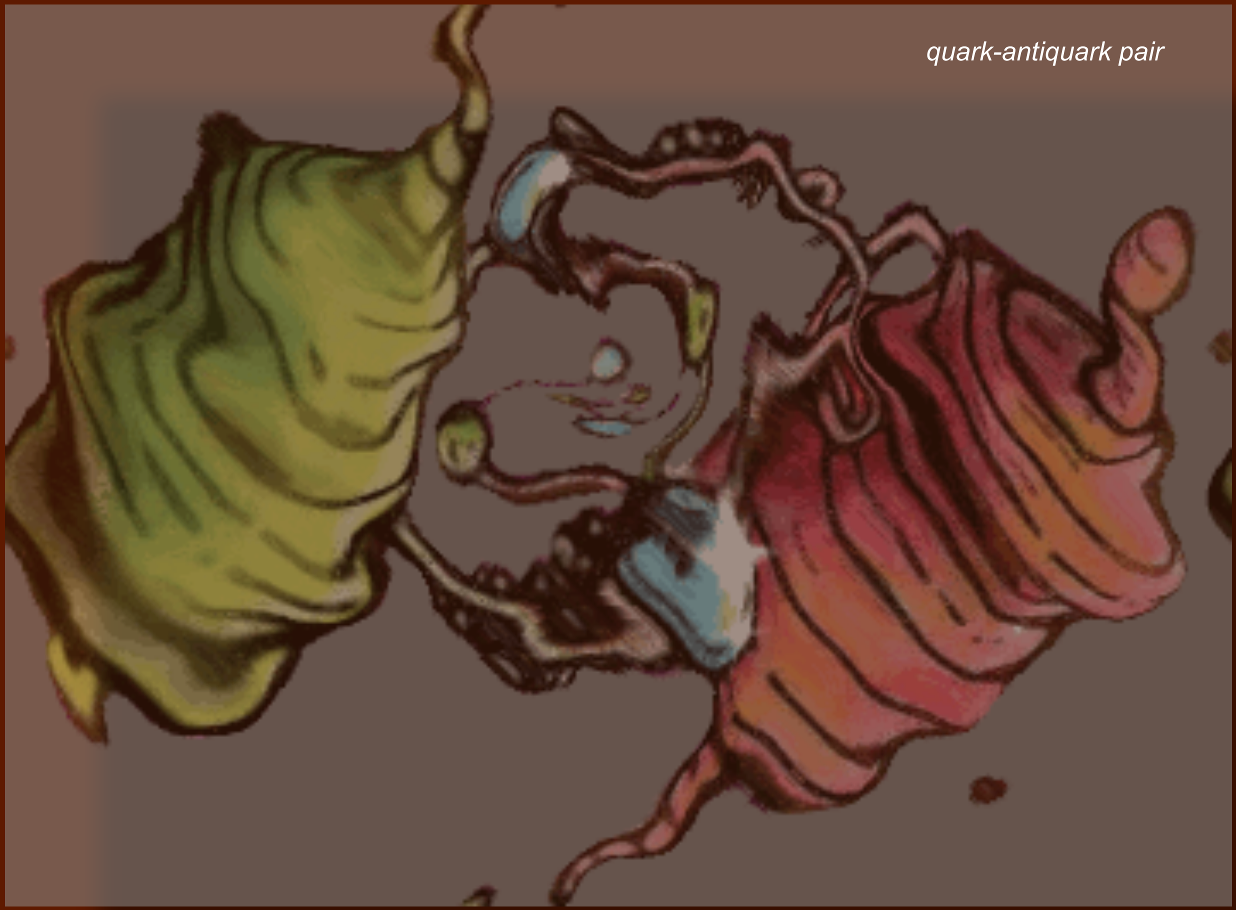
QCD is very successful in the asymptotic regime (< 1/10 fm):

- But, we have learned very little about hadron structure and its formation

Advances in QCD factorization in last 15 years:

- TMD factorization for two-scale observables,
- Collinear factorization for exclusive processes with a small t

Thank for your attention

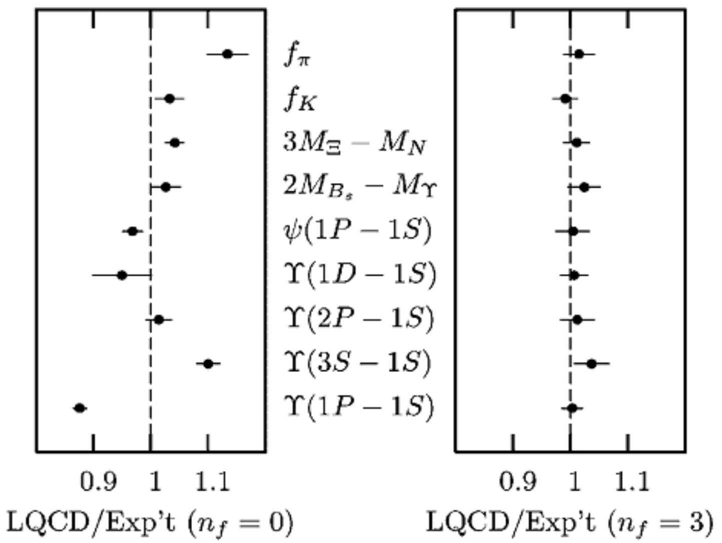


Oil paitings: Astrid Morreale (thanks Astrid!)

Backup slides

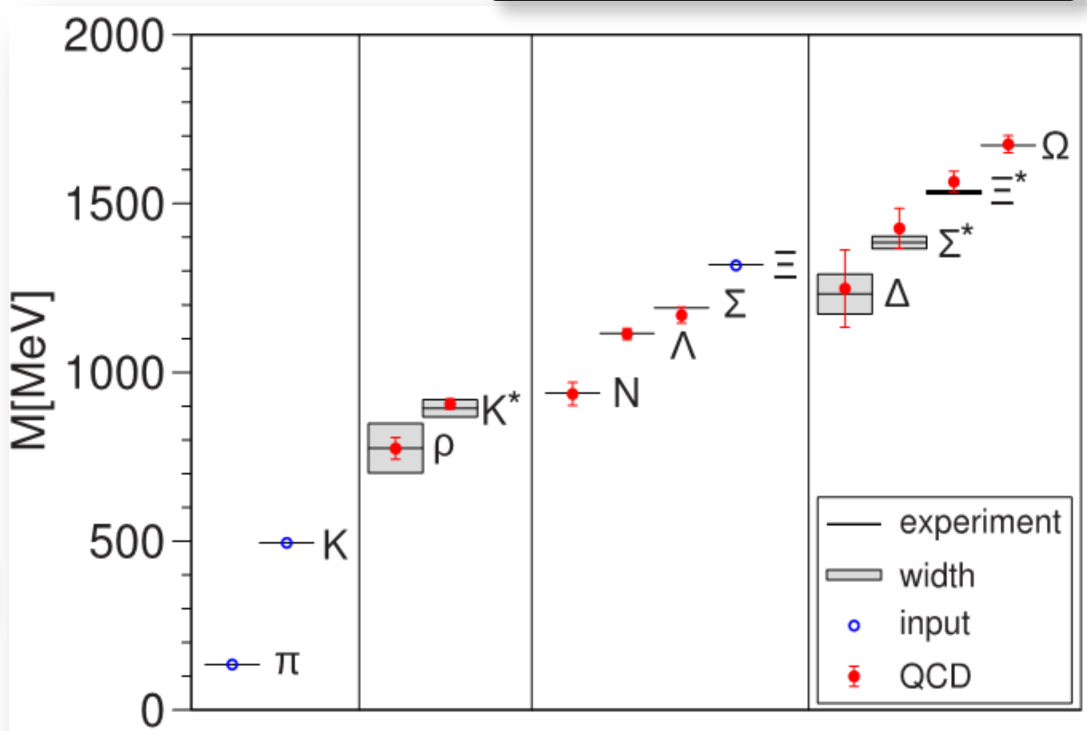
Lattice QCD “propaganda plots”

P. Haegler



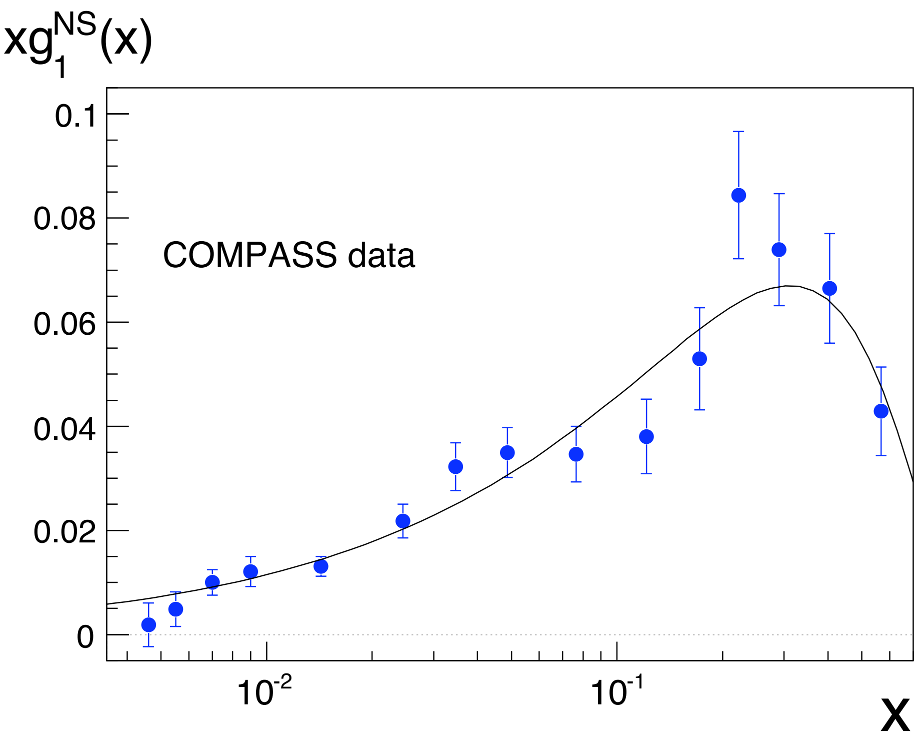
Davies, Lepage et al. [MILC] PRL 2004

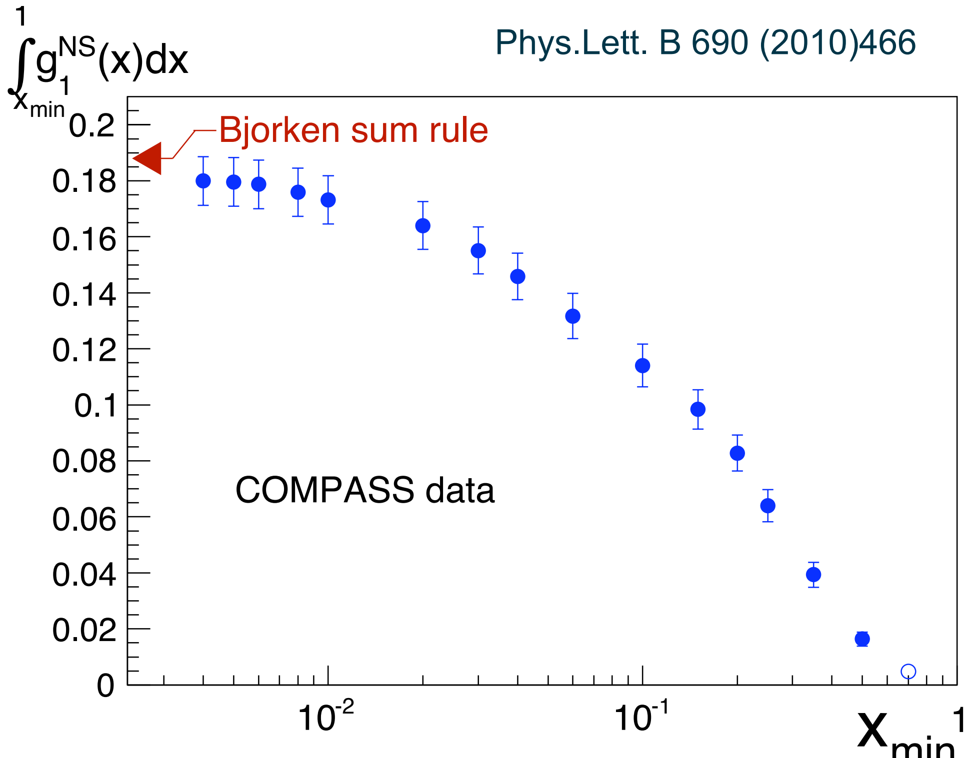
BMW (Dürr et al.) Nature 2009



with lattice pion masses down to m_p~190 MeV

COMPASS data, SMC, HERMES



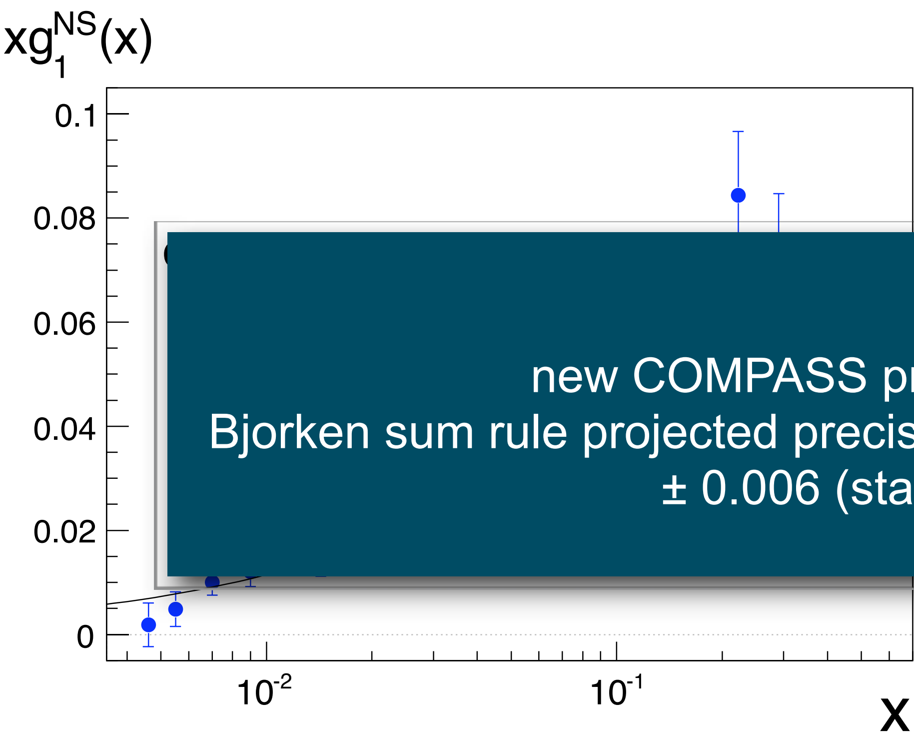


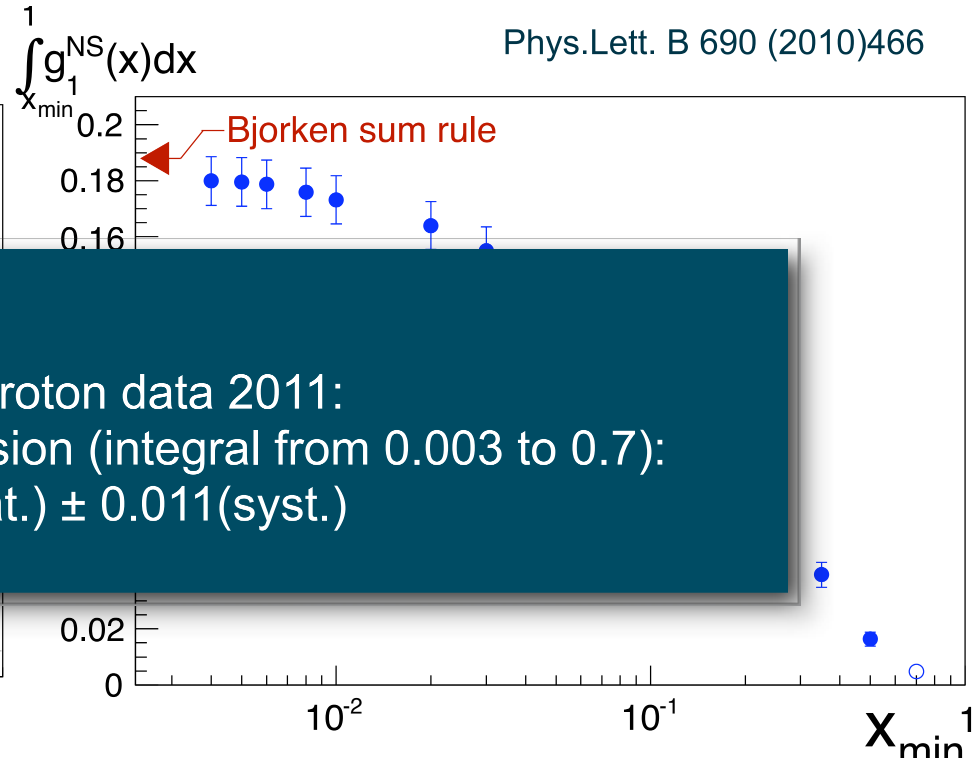
x range	Γ_1^{NS}
0 – 0.004	0.0098
0.004 – 0.7	$0.175 \pm \boxed{0.009 \pm 0.015}$
0.7 – 1.0	0.0048
0 – 1	$0.190 \pm 0.009 \pm 0.015$

in good agreement with SMC:
0.198 ± 0.023 ($Q^2 = 10$).

truncated moment from x=0.021:
$0.1583 \pm 0.0085 \pm 0.014$
in good agreement with HERMES:
$0.1484 \pm 0.0055 \pm 0.016$

COMPASS data, SMC, HERMES





new COMPASS proton data 2011:
Bjorken sum rule projected precision (integral from 0.003 to 0.7):
± 0.006 (stat.) ± 0.011(syst.)

x range	Γ_1^{NS}
0 – 0.004	0.0098
0.004 – 0.7	$0.175 \pm \boxed{0.009 \pm 0.015}$
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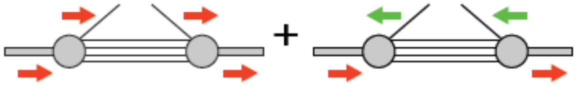
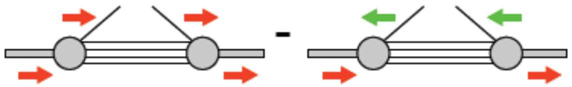
0.198 ± 0.023 (Q^2 =10).

truncated moment from x=0.021:
0.1583 ± 0.0085 ± 0.014
in good agreement with HERMES:
0.1484 ± 0.0055 ± 0.016

$q(x) \sim$

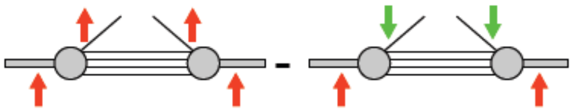


$\Delta q(x) \sim$



$\delta q(x) \sim$

... but in transverse basis ...

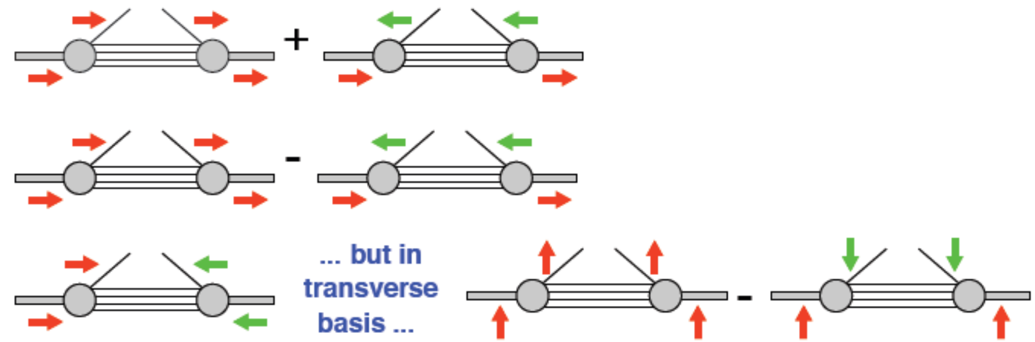


Properties of transversity

$q(x) \sim$

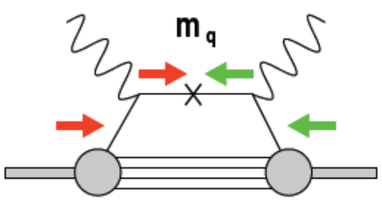
$\Delta q(x) \sim$

$\delta q(x) \sim$

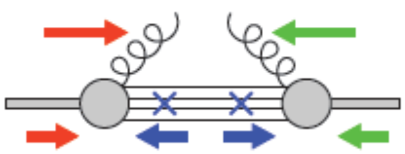


- **Chiral Odd**

  Helicity flip amplitudes occur only at $\mathcal{O}(m_q/Q)$ in inclusive DIS ...



- **No Gluons**



Angular momentum conservation: $\Lambda - \lambda = \Lambda' - \lambda'$

$\Rightarrow$ transversity has ***no gluon*** component

$\Rightarrow$ different Q^2 ***evolution*** than $\Delta q(x)$

A catalog of PDFs (II)

- $f_1(x, k_T^2)$: unpolarized. Integrated in k_T^2 gives the usual $f_1(x)$.

- $g_{1L}(x, k_T^2)$: longitudinally polarized. When integrated over k_T^2 it is the helicity function $g_1(x)$. From its 1st moment one can obtain $\Delta\Sigma = \Delta u + \Delta d + \Delta s$

  $\hookrightarrow$ COMPASS DIS and SIDIS results:
  PLB647(2007)8-17;
  PLB647(2007)330-340;
  PLB660(2008)458-465.

- $h_1(x, k_T^2)$: transversely polarized. When integrated over k_T^2 it survives, giving the transversity function $h_1(x)$.

  $\hookrightarrow$ COMPASS SIDIS results:
  PRL94(2005)202002;
  NPB765(2007)31-70;
  PLB673(2009)127-135.

A fly in the ointment

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  PLB673(2009)127-135.

A catalog of PDFs (III)

TMD PDFs - k_T dependent

- $f_{1T}^{\perp}(x, k_T^2)$: Sivers function. It describes the distortion of the probability distribution of a non-polarized quark when it is inside a transversely polarized nucleon.

  $\hookrightarrow$COMPASS DIS results:

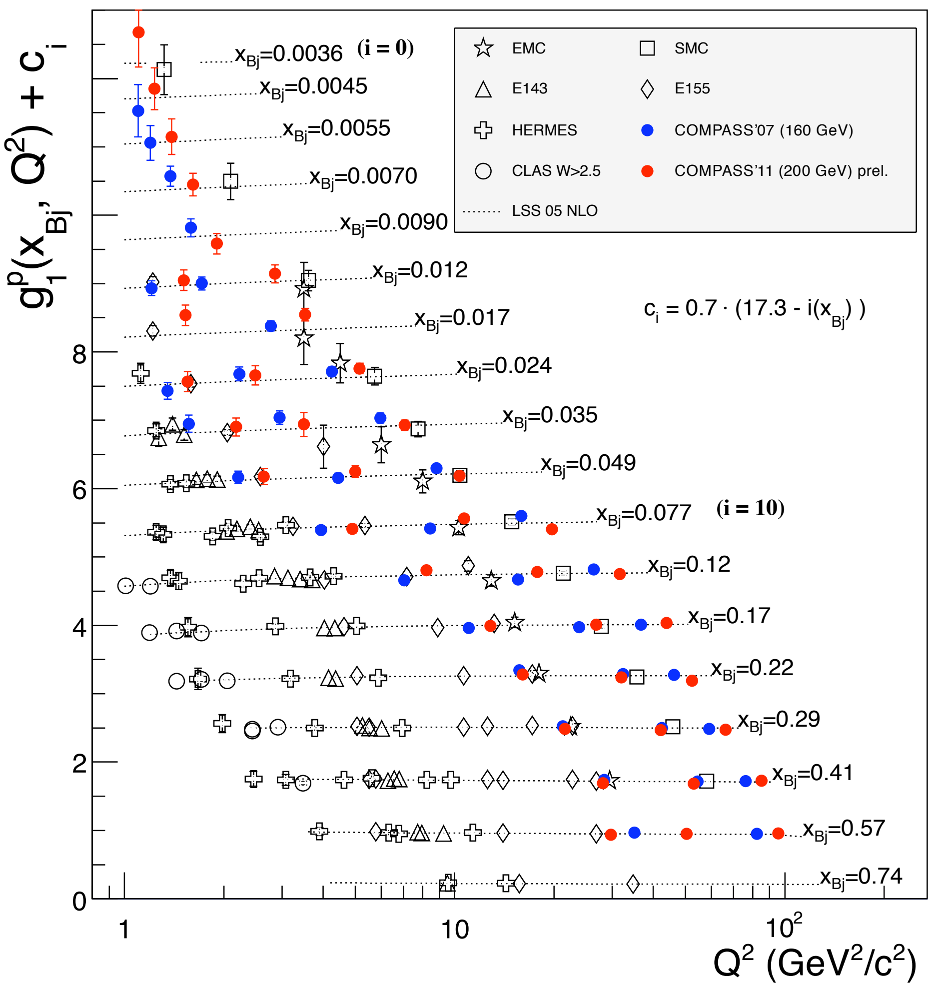
  PRL94(2005)202002;
  NPB765(2007)31-70;
  PLB673(2009)127-135.

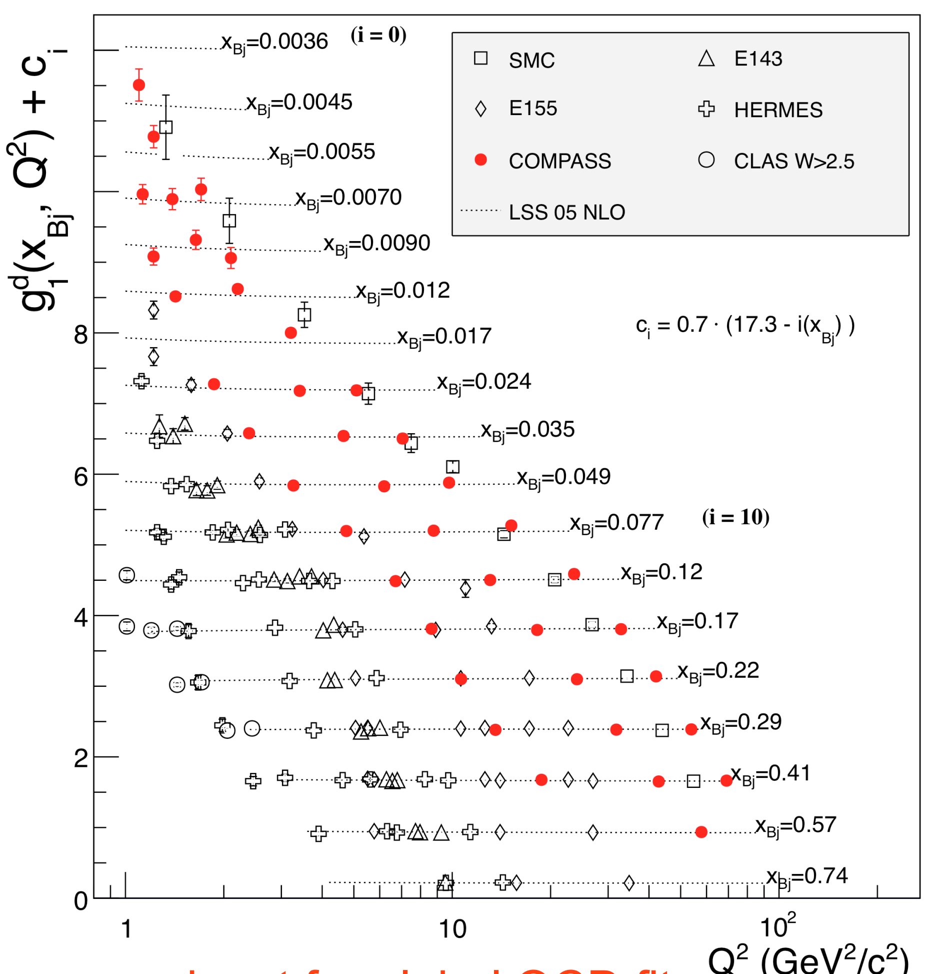
- $h_1^{\perp}(x, k_T^2)$: Boer-Mulders function. It describes the correlation between the transverse spin and the transverse momentum of a quark inside the unpolarized hadron.

- $h_{1T}^{\perp}(x, k_T^2)$: Pretzelosity function. It describes the transverse polarization of a quark, along its intrinsic k_T direction. It allows to access the orbital angular momentum information.

World data for proton and deuteron g_1 structure function

COMPASS proton data 2011@200 GeV included

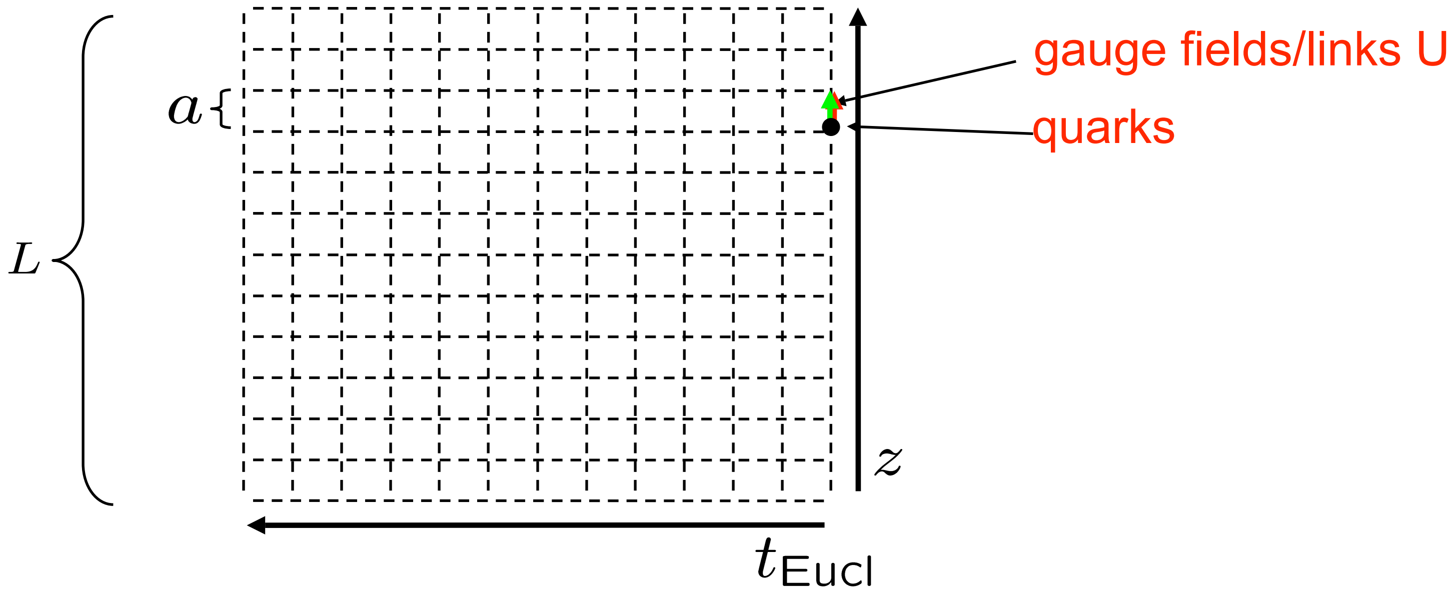




new input for global QCD fits

Lattice QCD calculations of hadron structure

P. Haegler



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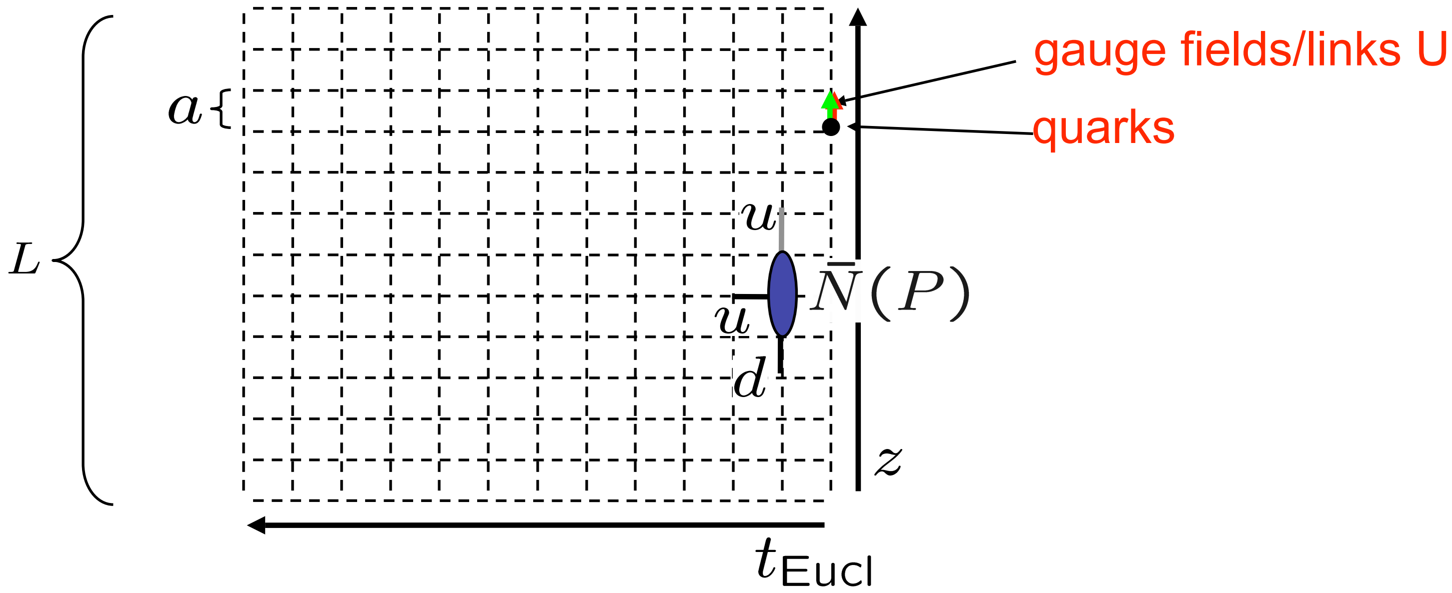

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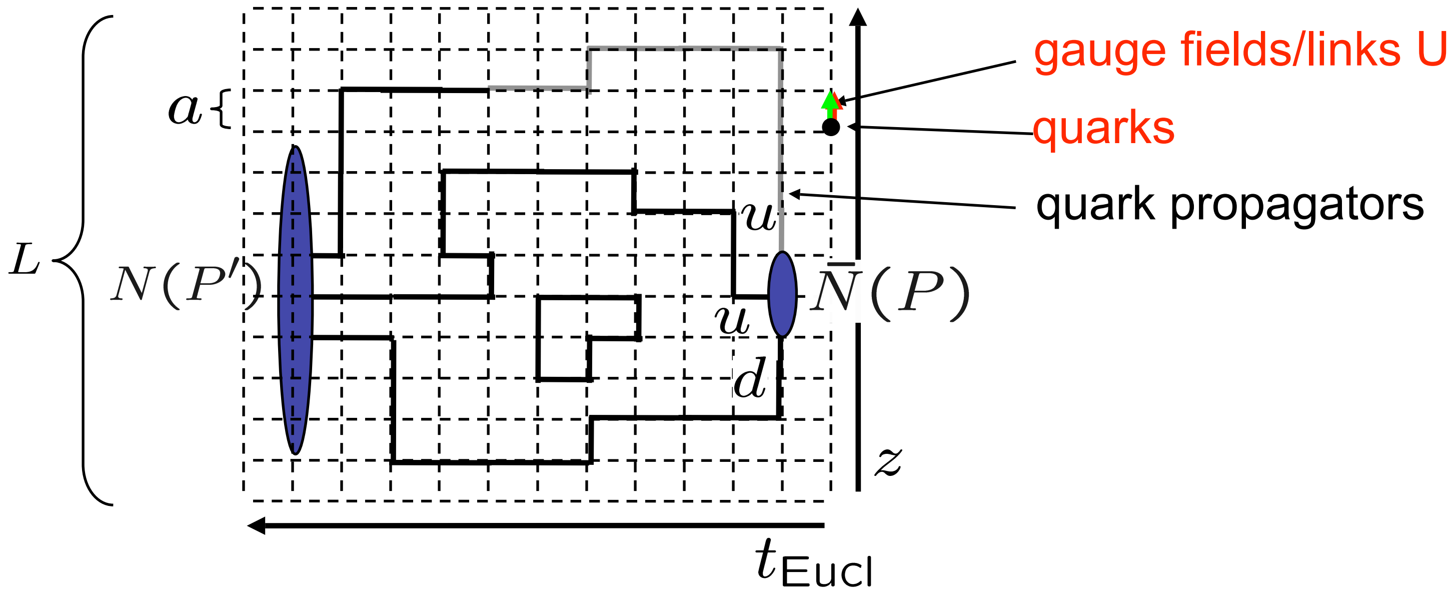

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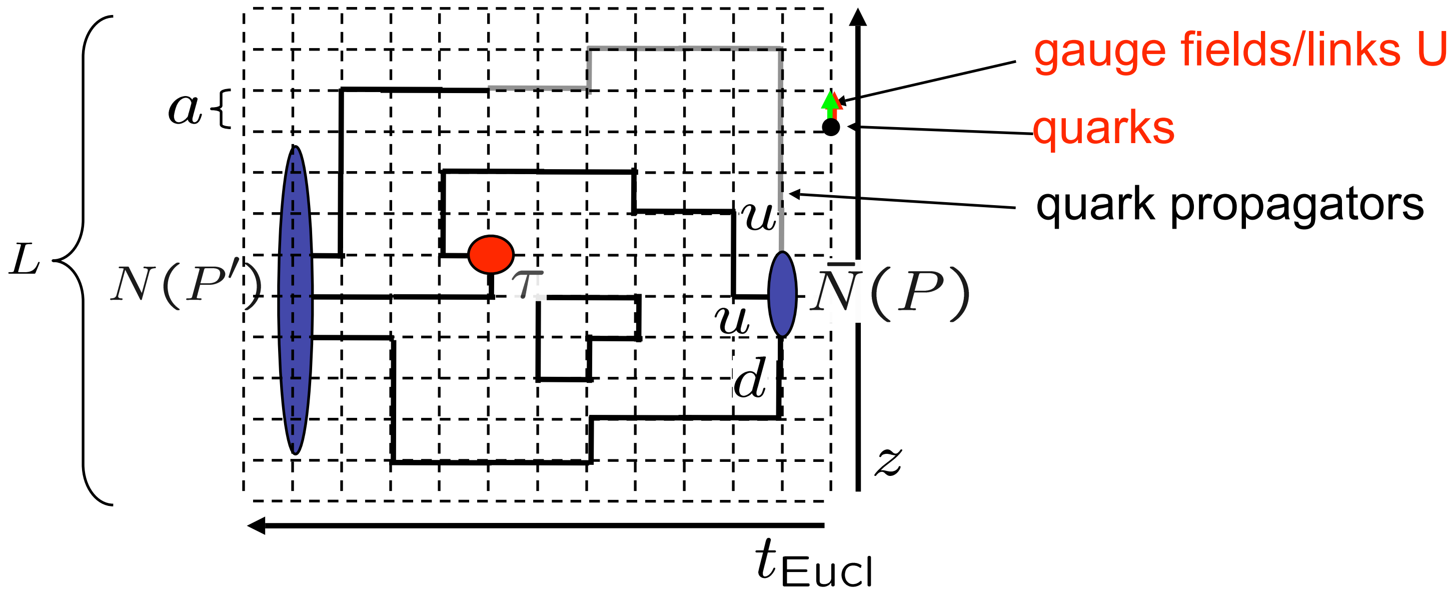
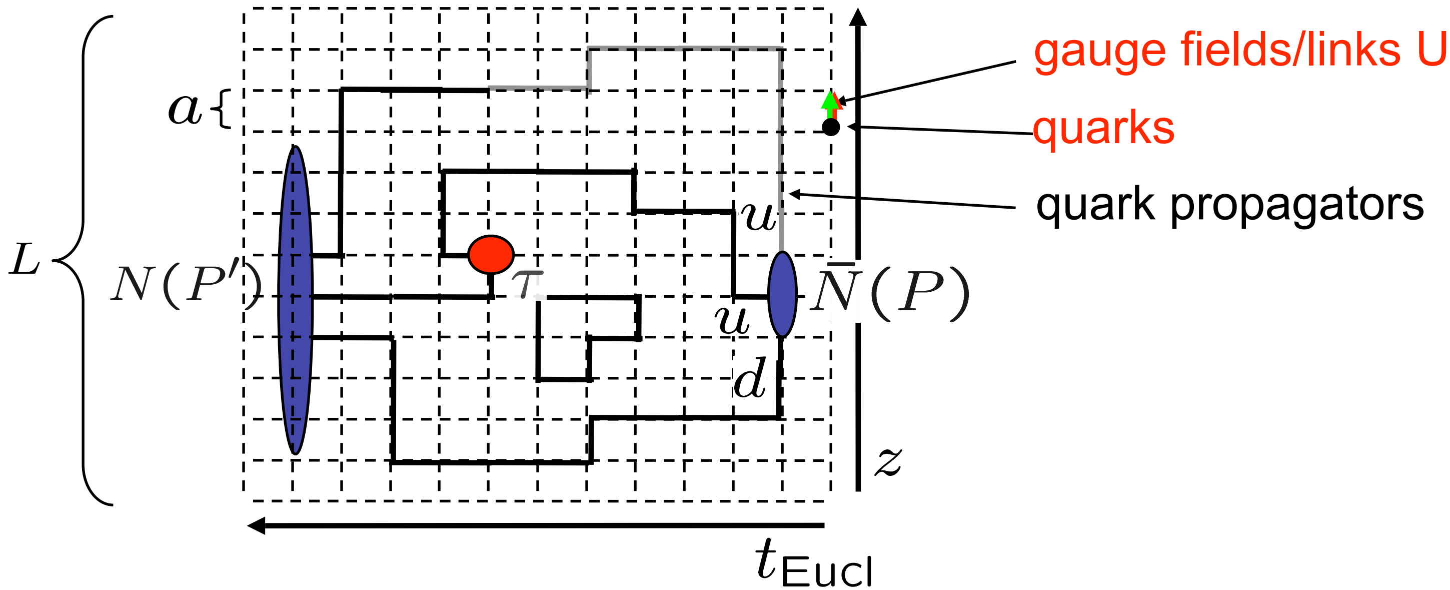


🔴 = vector-, axialvector-, *quark spin flip-*, (spin-2) *graviton-*, *„spin-n"* coupling

P. Haegler



● = vector-, axialvector-, *quark spin flip-,* (spin-2) *graviton-, „spin-n“* coupling

$$\langle q_2 \bar{q}_1 \rangle \propto \int DADqd\bar{q}e^{iS[q,\bar{q},A]} \rightarrow \left[\int DUe^{-S[U]} \det D[U]\right] D^{-1}_{1\rightarrow 2}[U] \approx \frac{1}{N}\sum_{i=1}^{N} D^{-1}_{1\rightarrow 2}[U_i]$$

compute the path–integral numerically

illustration: QED

$$\vec{J}_c = \int d^3x \psi^\dagger \vec{\gamma}\gamma_5 \psi + \int d^3x \psi^\dagger \left[\vec{x} \times (-i\vec{\nabla})\right] \psi + \int d^3x (\vec{E} \times \vec{A}) + \int d^3x E^i \left[\vec{x} \times \vec{\nabla} A^i\right]$$
$$= \vec{S}_c^e + \vec{L}_c^e + \vec{S}_c^\gamma + \vec{L}_c^\gamma. \quad (9.6)$$

The Bellifante TAM can be defined and decomposed as follows:

$$\vec{J}_B = \int d^3x \psi^\dagger \vec{\gamma}\gamma_5 \psi + \int d^3x \psi^\dagger \left[\vec{x} \times (-i\vec{D})\right] \psi + \int d^3x (\vec{E} \times \vec{B}$$
$$= \vec{S}_B^e + \vec{L}_B^e + \vec{J}_B^\gamma, \quad 9.7)$$

and finally Chen's proposition is:

$$\vec{J}_{Ch} = \int d^3x \psi^\dagger \vec{\gamma}\gamma_5 \psi + \int d^3x \psi^\dagger \left[\vec{x} \times (-i\vec{D}_{pure})\right] \psi + \int d^3x (\vec{E} \times \vec{A}_{phys}) + \int d^3x E^i \left[\vec{x} \times \vec{\nabla} A^i_{psyh}\right]$$
$$= \vec{S}_{Ch}^e + \vec{L}_{Ch}^e + \vec{S}_{Ch}^\gamma + \vec{L}_{Ch}^\gamma. \quad (9.8)$$

As usual $D^\mu = \partial^\mu - ieA^\mu$, E and B are electromagnetic fields. In the proposition by Chen and collaborators the photon (gluon) field is decomposed as $\vec{A} = \vec{A}_{phys} + \vec{A}_{pure}$ and:

$$\vec{\nabla}.\vec{A}_{phys} = 0, \qquad \vec{\nabla} \times \vec{A}_{pure} = 0. \quad (9.9)$$

Gluon helicity distribution

$$\Delta g = \int_0^1 \mathrm{d}x\, \Delta g(x)$$
$$= \int_0^1 \mathrm{d}x\, \frac{i}{xP^+} \int \frac{\mathrm{d}z^-}{2\pi}\, e^{ixP^+z^-} \langle P,\Lambda|2\mathrm{Tr}[F^{+\alpha}(0)\mathcal{W}_{[0,z^-]}\tilde{F}^+{}_\alpha(z^-)\mathcal{W}_{[z^-,0]}]|P,\Lambda\rangle$$
$$= \frac{\epsilon^{+-}{}_{\alpha\beta}}{2P^+} \langle P,\Lambda|2\mathrm{Tr}[F^{+\alpha}(0) \int \mathrm{d}z^-\, \tfrac{1}{2}\epsilon(z^-)\, \mathcal{W}_{[0,z^-]}F^{+\beta}(z^-)\mathcal{W}_{[z^-,0]}]|P,\Lambda\rangle$$

« Measurable », gauge invariant but non-local

Light-front gauge $\boxed{A^+ = 0}$

[Jaffe-Manohar (1990)]

$$= \frac{\epsilon^{+-}{}_{\alpha\beta}}{2P^+} \langle P,\Lambda| - 2\mathrm{Tr}[F^{+\alpha}(0)A^\beta(0)]|P,\Lambda\rangle$$
$$= \frac{1}{2P^+} \langle P,\Lambda|\tfrac{1}{2}\epsilon^{+-}{}_{\alpha\beta}\, M^{+\alpha\beta,\mathrm{JM}}_{g,\mathrm{spin}}(0)|P,\Lambda\rangle$$
$$= \frac{\langle P,\Lambda|S^{z,\mathrm{JM}}_g|P,\Lambda\rangle}{\langle P,\Lambda|P,\Lambda\rangle}$$

Light-front GIE $\boxed{A^+_{\mathrm{phys}} = 0}$

[Hatta (2011)]

$$= \frac{\epsilon^{+-}{}_{\alpha\beta}}{2P^+} \langle P,\Lambda| - 2\mathrm{Tr}[F^{+\alpha}(0)A^{\beta,\mathrm{Hatta}}_{\mathrm{phys}}(0)]|P,\Lambda\rangle$$
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Orbital angular momentum

E.Leader, Phys.Rev.D83, 096012 (2011)

Brief comments on the gluon/gamma spin

The spin $S_{can}(\gamma)$ is not gauge invariant.

The Bellinfante $J_{bel}(\gamma)$ is gauge invariant, but does not split into orbital and spin parts.

It is generally stated that there is no gauge invariant way of splitting $J(\gamma)$ into spin and orbital parts.

Chen et al disagree, but the object $S_{chen}(\gamma)$ they produce really does not make sense as a spin vector.
The spin density at a point x depends on the fields throughout all space!

Orbital angular momentum

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Th
spli

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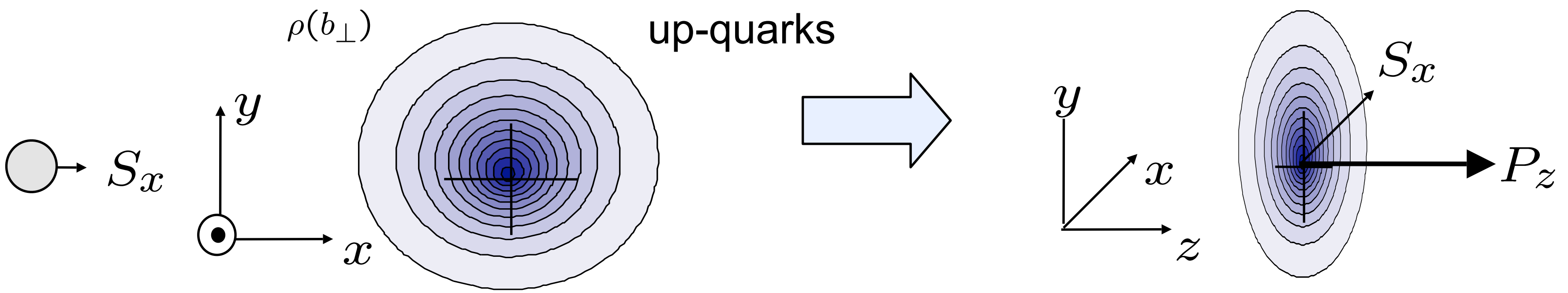
way

Decomposition is useful if, only if, each term can be related to physical observables with controllable approximation

Chen et al disagree, but the object $S_{chen}(\gamma)$ they produce really does not make sense as a spin vector.
The spin density at a point x depends on the fields throughout all space!

Sivers function and GPD

M.Burhardt 2002/2003



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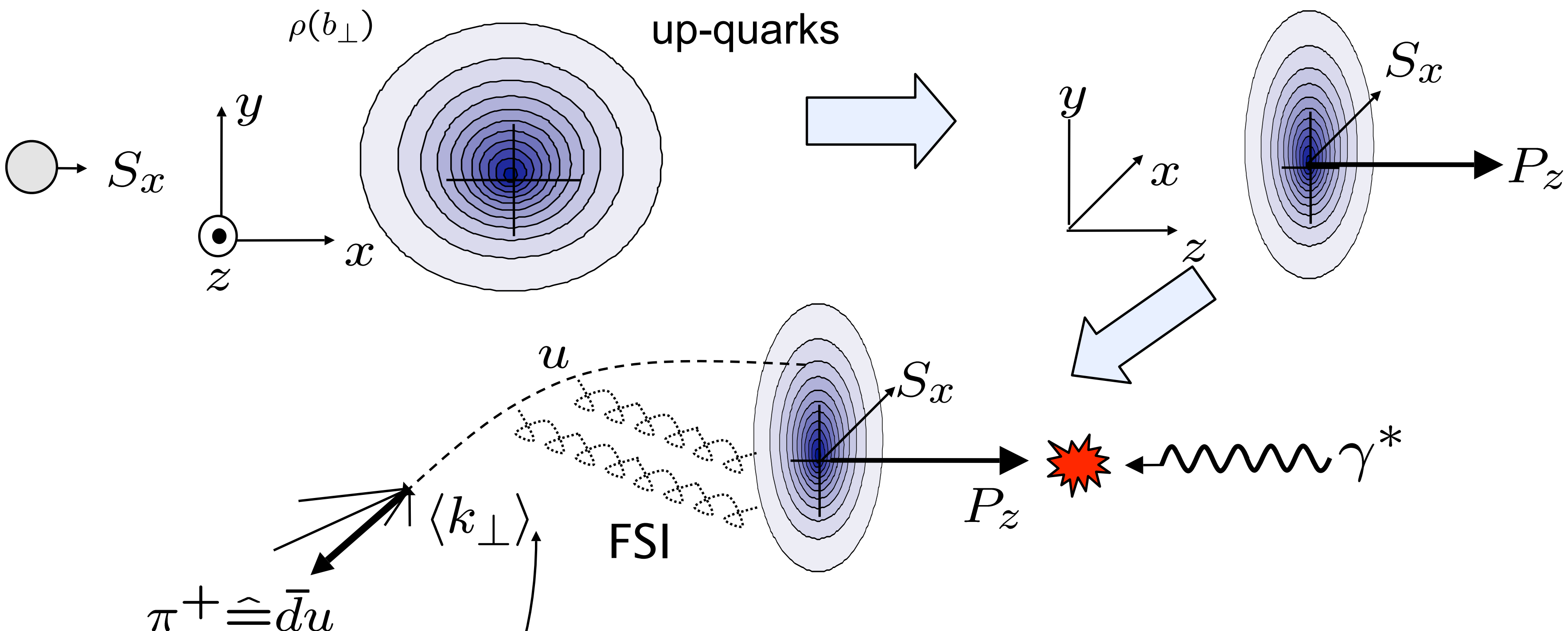


Sivers–asymmetry

„chromodynamic lensing"

$$\langle k_\perp \rangle \propto \int d^2k_\perp k_y^2 f_{1T}^{\perp}(k_\perp^2) \longleftrightarrow \int d^2b_\perp \tilde{\mathcal{I}}_\perp(b_\perp) \overbrace{\langle P^+, 0_\perp | \bar{q}(b_\perp)\gamma^+ q(b_\perp) | P^+, 0_\perp \rangle}^{\rho(b_\perp)}$$

Difference asymmetry

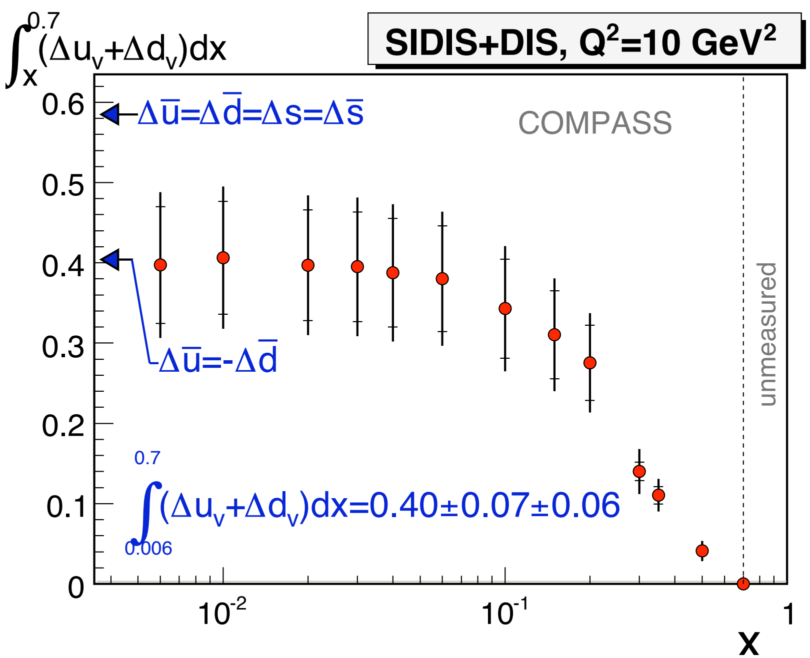
Idea: Phys.Lett.B230(1989)141,
SMC:Phys.Lett.B369(1996)93,
COMPASS: Phys.Lett.B660(2008)458

$$A^{+-} = \frac{(\sigma^{h^+}_{\uparrow\downarrow} - \sigma^{h^-}_{\uparrow\downarrow}) - (\sigma^{h^+}_{\uparrow\uparrow} - \sigma^{h^-}_{\uparrow\uparrow})}{(\sigma^{h^+}_{\uparrow\downarrow} - \sigma^{h^-}_{\uparrow\downarrow}) + (\sigma^{h^+}_{\uparrow\uparrow} - \sigma^{h^-}_{\uparrow\uparrow})}$$

$$A_d^{\pi^+-\pi^-}(x) = A_d^{K^+-K^-}(x) = \frac{\Delta u_v(x) + \Delta d_v(x)}{u_v(x) + d_v(x)} \rightarrow$$

Only valence quarks!

Fragmentation functions cancel out in LO and under the assuption of independent fragmentation.



$$\Delta\bar{u} = \Delta\bar{d} = \Delta s = \Delta\bar{s}$$ symmetric scenario

$$\Delta\bar{u} = -\Delta\bar{d}$$ asymmetric scenario

$$\Gamma_v = \int_0^1 (\Delta u_v(x) + \Delta d_v(x))dx$$

$$\Delta\bar{u} + \Delta\bar{d} = 3\Gamma_1^N - \tfrac{1}{2}\Gamma_v + \tfrac{1}{12}a_8 = (\Delta s + \Delta\bar{s}) + \tfrac{1}{2}(a_8 - \Gamma_v)$$

Spin degree of freedom

- Spin is a quantum and relativistic object, originated from space-time symmetry and survives high-energy limit
- Spin plays a critical role in determining the basic structure of fundamental interactions
- Spin provides a unique opportunity to probe the inner structure of a composite system such as the nucleon

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"Experiments with spin have killed more theories than any other single physical parameter"

Elliot Leader, Spin in Particle Physics, Cambridge U. Press (2001)

"Polarisation data has often been the graveyard of fashionable theories. If theorists had their way they might well ban such measurements altogether out of selfprotection."

J. D. Bjorken, Proc. Adv. Research Workshop on QCD Hadronic Processes St. Croix, Virgin Islands (1987).

First moment of g_1 structure functions

Sum rules

first moment $\Gamma_1 = \int g_1(x)dx$

Bjorken s.r. $\Gamma_1^p - \Gamma_1^n = \frac{g_A}{6g_V} C_1^{NS}$

Ellis-Jaffe s.r. $\Gamma_1^{p,n} = \left(\pm a_3 + \frac{a_8}{3} \right) \frac{C_1^{NS}}{12} + a_0 \frac{C_1^S}{9}$

$a_3, a_8, g_{A,V}$ measured in weak β decays (+Su(3)$_f$)

$C_1^{S,NS}$ calculable in pQCD

$$a_0 = \Delta\Sigma = \Delta u + \Delta d + \Delta s$$

Quark contribution to nucleon helicity

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from Y. Goto *et al.*, PRD62 (2000) 034017:
(SU(3)$_f$ assumed for weak decays)
$a_8 = 0.585 \pm 0.025$

$$a_{0|Q_0^2=3(GeV/c)^2} = 0.35 \pm 0.03(stat) \pm 0.05(syst)$$

QCD NLO

$$\hat{a}_{0|Q^2\to\infty} = 0.33 \pm 0.03(stat) \pm 0.05(syst)$$

beyond NLO

C_1 calculated behind 3 loops app.
S.A.Larin *et al.*,Phys.Lett.B404(1997)153

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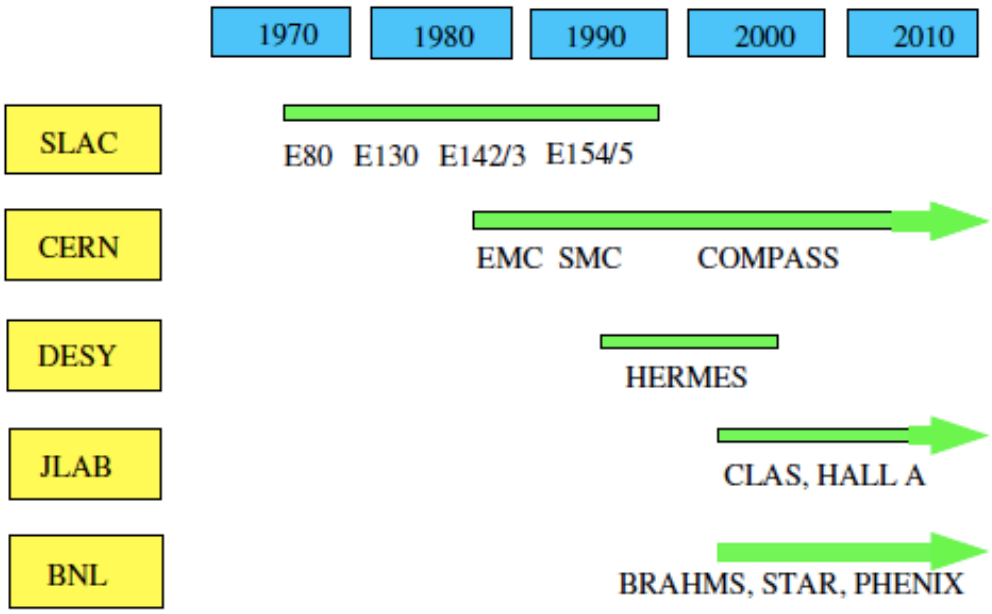
S.A.Larin *et al.*,Phys.Lett.B404(1997)153

$$\boxed{(\Delta s + \Delta\bar{s}) = \frac{1}{3}(\hat{a}_0 - a_8) = -0.08 \pm 0.01(stat) \pm 0.02(syst)}$$

Polarised experiments

- Inclusive spin-dependent DIS
  EMC, SMC, COMPASS (CERN),E142,E143,E154,E156 (SLAC),
  HERMES (DESY), CLASS, HALL-A (J LAB)
- Semi-inclusive DIS: SMC, COMPASS,HERMES
- Polarized pp collision RHIC-PHENIX & STAR, BRAHMS (Brookhaven)
- ee: BELLE (KEK) (Fragmentation functions)



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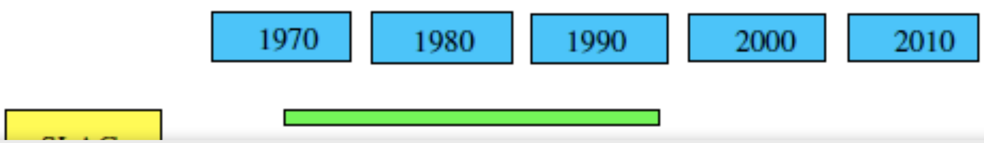

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Nucleon spin decomposition

$$\tfrac{1}{2} = \tfrac{1}{2}\Delta\Sigma + \Delta G + L \longrightarrow J?$$

$$\frac{1}{2} = \frac{1}{2}\sum_{q,\bar{q}} \int dx \cdot \Delta_T q(x) + \sum_{q,\bar{q},g} \langle L_z \rangle$$

naive scenario - $\Delta\Sigma=1$, the rest is 0 (e.g. SU(6) static model)
relativistic corrections change 1 to ~0.65 for longitudinal
and ~0.85 for transversely polarised target

Transversely polarized target

1. No gluons! (angular momentum conservation - different QCD evolution)
2. Transversity structure function - C-odd and chiral-odd. No accessible in DIS but measured in DY or SIDIS processes thanks to Collins FF

GPDs and anomalous magnetic moments

The E functions are inaccessible in DIS as the DIS is described by forward limit. However the limit: $t = 0$ and $\xi = 0$ of E exists:

$$E^i(Q^2, x, 0, 0) = \kappa^i(x)$$

and its integral leads to the anomalous magnetic moment of the nucleon:

$$\sum_q e_q \int_0^1 (\kappa^q(x) - \kappa^{\bar{q}}(x))dx = \kappa_N$$

$$\begin{aligned} J^q(Q^2) &= \frac{1}{2}\sum_q \int_0^1 x(q(x,Q^2) + \bar{q}(x,Q^2) + \kappa^q(x,Q^2) + \kappa^{\bar{q}}(x,Q^2))dx \\ J^G(Q^2) &= \frac{1}{2}\int_0^1 x(G(x,Q^2) + \kappa^G(x,Q^2))dx \end{aligned}$$

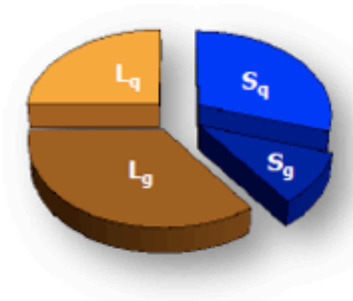
Interesting constrains

Angular momentum

Canonical

$$\vec{p} = \frac{\partial L}{\partial \vec{v}}$$

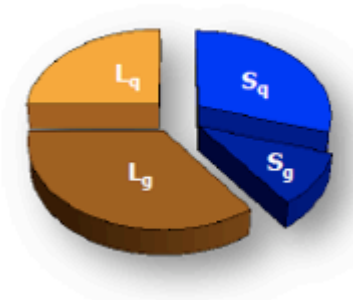
[Jaffe-Manohar (1990)]



$$\vec{S}_q = \frac{1}{2}\int d^3r\,\psi^\dagger\vec{\Sigma}\psi$$
$$\vec{L}_q = \int d^3r\,\psi^\dagger\vec{r}\times(-i\vec{\nabla})\psi$$
$$\vec{S}_g = \int d^3r\,\vec{E}^a\times\vec{A}^a$$
$$\vec{L}_g = \int d^3r\,E^{ai}\vec{r}\times\vec{\nabla}A^{ai}$$

Gauge non-invariant!

[Chen *et al.* (2008)] $\boxed{A = A_{\rm pure} + A_{\rm phys}}$



$$\vec{S}_q = \frac{1}{2}\int d^3r\,\psi^\dagger\vec{\Sigma}\psi$$
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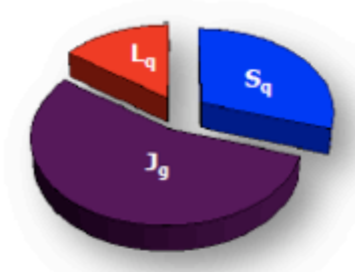
$$\vec{L}_g = \int d^3r\,E^{ai}\vec{r}\times\vec{D}_{\rm pure}A^{ai}_{\rm phys}$$

Gauge-invariant extension (GIE)

Kinetic

$$\vec{\pi} = m\vec{v} = \vec{p} + g\vec{A}$$

[Ji (1997)] $\boxed{\vec{D} = \vec{\nabla} + ig\vec{A}}$



$$\vec{S}_q = \frac{1}{2}\int d^3r\,\psi^\dagger\vec{\Sigma}\psi$$
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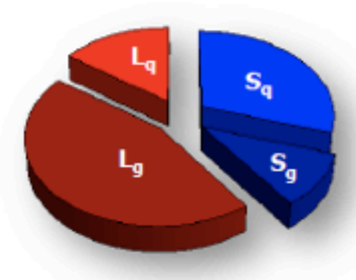
$$\vec{J}_g = \int d^3r\,\vec{r}\times(\vec{E}^a\times\vec{B}^a)$$

[Wakamatsu (2010)] $\boxed{A = A_{\rm pure} + A_{\rm phys}}$



$$\vec{S}_q = \frac{1}{2}\int d^3r\,\psi^\dagger\vec{\Sigma}\psi$$
$$\vec{L}_q = \int d^3r\,\psi^\dagger\vec{r}\times(-i\vec{D})\psi$$
$$\vec{S}_g = \int d^3r\,\vec{E}^a\times\vec{A}^a_{\rm phys}$$
$$\vec{L}_g = \int d^3r\,\vec{r}\times(\vec{E}^a\times\vec{B}^a) - \int d^3r\,\vec{E}^a\times\vec{A}^a_{\rm phys}$$

Anomaly and the role of gluons

"Switching on" spin leads us to two complications:

$$q^{-} \sim \psi \tfrac{1}{2}(1-\gamma_5)\gamma_\mu \psi \qquad q^{+} \sim \psi \tfrac{1}{2}(1+\gamma_5)\gamma_\mu \psi$$

$$\Delta q = u^{+} - u^{-} \sim \psi \tfrac{1}{2}\gamma_\mu \gamma_5 \psi$$

- axial current is not conserved due to Adler-Bell-Jackiw triangle anomaly
- there is no local, gauge invariant dimension-3 axial operator for gluons

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Question is if we need conserved current?

$\overline{\text{MS}}$ scheme - no but a_0 depends on the scale
AB scheme - yes - a_0 does not depend on the scale but now

$$a_0 = \Delta\Sigma - \tfrac{3}{2}\tfrac{\alpha_S}{\pi}\Delta G$$

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$\triangle$ anomaly gives a possible interpretation of the measured value of a_0
- if gluon polarization is large enough then E-J sum rule can be restored and quark contribution to the nucleon spin is significant as expected in simple QPM

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A catalog of PDFs

SIDIS cross section decomposition LO

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,d\phi_h\,dz_h\,dP_{hT}^2} = \frac{e^4}{32\pi^2 x Q^2}\frac{y}{1-\varepsilon}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon\, F_{UU,L} + \varepsilon\, cos(2\phi_h)F_{UU}^{cos2\phi_h}$$
$$+ \sqrt{2\varepsilon(1-\varepsilon)}\left(cos\phi_h F_{UU}^{cos\phi_h} + h_l sin\phi_h F_{LU}^{sin\phi_h}\right)$$
$$+ S_{\|}\left[\sqrt{2\varepsilon(1+\varepsilon)} sin\phi_h F_{UL}^{sin\phi_h} + \varepsilon\, sin(2\phi_h)F_{UL}^{sin2\phi_h}\right]$$
$$+ S_{\|}h_l\left[\sqrt{1-\varepsilon^2}F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} cos\phi_h F_{LL}^{cos\phi_h}\right]$$
$$+ |S_\perp|\Big[sin(\phi_h-\phi_S)\left(F_{UT,T}^{sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{sin(\phi_h-\phi_S)}\right)$$
$$+ \varepsilon\, sin(3\phi_h-\phi_S)F_{UT}^{sin(3\phi_h-\phi_S)} + \varepsilon\, sin(\phi_h+\phi_S)F_{UT}^{sin(\phi_h+\phi_S)}$$
$$+ \sqrt{2\varepsilon(1+\varepsilon)}\left(sin\phi_S F_{UT}^{sin\phi_S} + sin(2\phi_h-\phi_S)F_{UT}^{sin(2\phi_h-\phi_S)}\right)\Big]$$
$$+ |S_\perp|h_l\Big[\sqrt{1-\varepsilon^2}cos(\phi_h-\phi_S)F_{LT}^{cos(\phi_h-\phi_S)}$$
$$+ \sqrt{2\varepsilon(1-\varepsilon)}\left(cos\phi_S F_{LT}^{cos\phi_S} + cos(2\phi_h-\phi_S)F_{LT}^{cos(2\phi_h-\phi_S)}\right)\Big]\Big\},$$

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$$+\sqrt{2\varepsilon(1-\varepsilon)}\left(cos\phi_h F_{UU}^{cos\phi_h} + h_l sin\phi_h F_{LU}^{sin\phi_h}\right)$$

Cahn & Boer-Mulders

unpolarised target

$$+S_{\parallel}\left[\sqrt{2\varepsilon(1+\varepsilon)} sin\phi_h F_{UL}^{sin\phi_h} + \varepsilon\, sin(2\phi_h) F_{UL}^{sin2\phi_h}\right]$$

$$+S_{\parallel} h_l\left[\sqrt{1-\varepsilon^2} F_{LL} + \sqrt{2\varepsilon(1-\varepsilon)} cos\phi_h F_{LL}^{cos\phi_h}\right]$$

longitudinaly polarised target

$$+|S_\perp|\Big[sin(\phi_h-\phi_S)\left(F_{UT,T}^{sin(\phi_h-\phi_S)} + \varepsilon\, F_{UT,L}^{sin(\phi_h-\phi_S)}\right)$$

$$+\varepsilon\, sin(3\phi_h-\phi_S) F_{UT}^{sin(3\phi_h-\phi_S)} + \varepsilon\, sin(\phi_h+\phi_S) F_{UT}^{sin(\phi_h+\phi_S)}$$

$$+\sqrt{2\varepsilon(1+\varepsilon)}\left(sin\phi_S F_{UT}^{sin\phi_S} + sin(2\phi_h-\phi_S) F_{UT}^{sin(2\phi_h-\phi_S)}\right)\Big]$$

$$+|S_\perp| h_l\Big[\sqrt{1-\varepsilon^2} cos(\phi_h-\phi_S) F_{LT}^{cos(\phi_h-\phi_S)}$$

$$+\sqrt{2\varepsilon(1-\varepsilon)}\left(cos\phi_S F_{LT}^{cos\phi_S} + cos(2\phi_h-\phi_S) F_{LT}^{cos(2\phi_h-\phi_S)}\right)\Big]\Big\},$$

transversely polarised target

Sivers
Collins
Pretzelosity
Worm Gear

A catalog of PDFs

SIDIS cross section decomposition LO

$$\frac{d\sigma}{dx\,dy\,d\phi_S\,d\phi_h\,dz_h\,dP_{hT}^2} = \frac{e^4}{32\pi^2 x Q^2}\frac{y}{1-\varepsilon}\left(1+\frac{\gamma^2}{2x}\right)\Big\{F_{UU,T} + \varepsilon\, F_{UU,L} + \varepsilon\, cos(2\phi_h) F_{UU}^{cos2\phi_h}$$

$$+\sqrt{2\varepsilon(1-\varepsilon)}\left(cos\phi_h F_{UU}^{cos\phi_h} + h_l sin\phi_h F_{LU}^{sin\phi_h}\right)$$

Cahn & Boer-Mulders

unpolarised target

$$+ S_{\|}\left[\sqrt{2\varepsilon(1+\varepsilon)} sin\phi_h F_{UL}^{sin\phi_h} + \varepsilon\, sin(2\phi_h) F_{UL}^{sin2\phi_h}\right]$$

...get

...sely
...d target

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...Gear

$$F_{UU,T} \sim \sum_q e_q^2 \cdot f_1^q \otimes D_q^h, \qquad F_{LT}^{cos(\phi_h-\phi_S)} \sim \sum_q e_q^2 \cdot g_{1T}^q \otimes D_q^h,$$

$$F_{LL} \sim \sum_q e_q^2 \cdot g_{1L}^q \otimes D_q^h, \qquad F_{UT,T}^{sin(\phi_h-\phi_S)} \sim \sum_q e_q^2 \cdot f_{1T}^{\perp q} \otimes D_q^h,$$

$$F_{UU}^{cos2\phi_h} \sim \sum_q e_q^2 \cdot h_1^{\perp q} \otimes H_1^{\perp q}, \qquad F_{UT}^{sin(\phi_h+\phi_S)} \sim \sum_q e_q^2 \cdot h_1^q \otimes H_1^{\perp q},$$

$$F_{UL}^{sin2\phi_h} \sim \sum_q e_q^2 \cdot h_{1L}^{\perp q} \otimes H_1^{\perp q}, \qquad F_{UT}^{sin(3\phi_h-\phi_S)} \sim \sum_q e_q^2 \cdot h_{1T}^{\perp q} \otimes H_1^{\perp q}.$$

1-D nucleon structure - QPM
well-established, simple, working!

$$\sigma \sim F_1(x) = \tfrac{1}{2}\sum_i e_q^2 q_i(x) \qquad F_2(x) \approx 2xF_1$$

QCD Collinear factorization for cross sections with one (e.g. Q^2) large momentum transfer

Gottfried sum rule:

$$S_G = \int_0^1 [(F_2^p(x) - F_2^n(x))/x]\,dx$$
$$= \frac{1}{3} + \frac{2}{3}\int_0^1 (\bar{u}_p(x) - \bar{d}_p(x))\;dx$$
$$= \frac{1}{3} \qquad (if\ \bar{u}_p = \bar{d}_p)$$