# Confronting BFKL dynamics with experimental studies of Mueller-Navelet jets at the LHC

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in collaboration with
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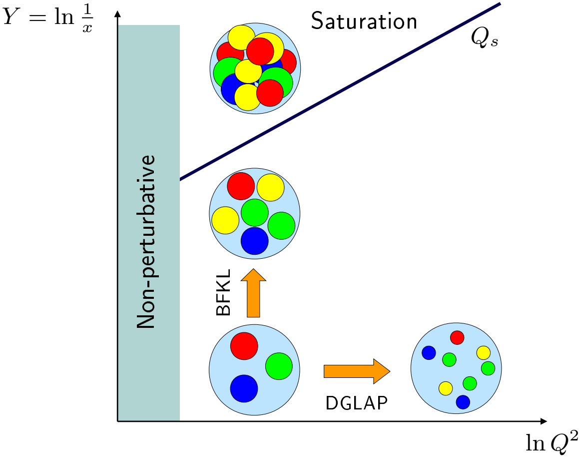
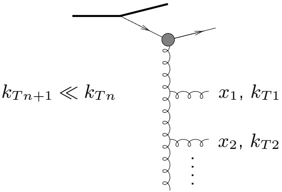
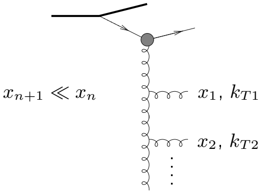
B. Ducloué, LS, S. Wallon, JHEP **1305** (2013) 096 [arXiv:1302.7012 [hep-ph]]
B. Ducloué, LS, S. Wallon, PRL **112** (2014) 082003 [arXiv:1309.3229 [hep-ph]]

# The different regimes of QCD



Small values of $\alpha_s$ (perturbation theory applies if there is a hard scale) can be compensated by large logarithmic enhancements.

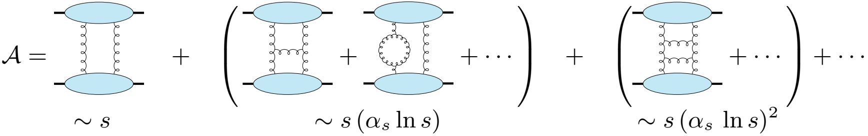
| DGLAP | BFKL |
|---|---|
|  |  |
| strong ordering in $k_T$ | strong ordering in $x$ |
| $\sum(\alpha_s \ln Q^2)^n$ | $\sum(\alpha_s \ln s)^n$ |



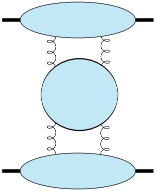
When $\sqrt{s}$ becomes very large, it is expected that a BFKL description is needed to get accurate predictions

QCD in the perturbative Regge limit

The amplitude can be written as:




this can be put in the following form :



← Impact factor

← Green's function

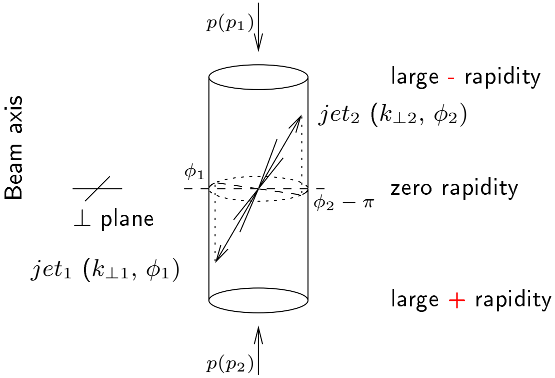
← Impact factor

- Higher order corrections to BFKL kernel are known at NLL order (Lipatov Fadin; Camici, Ciafaloni), now for arbitrary impact parameter $\alpha_S \sum_n (\alpha_S \ln s)^n$ resummation

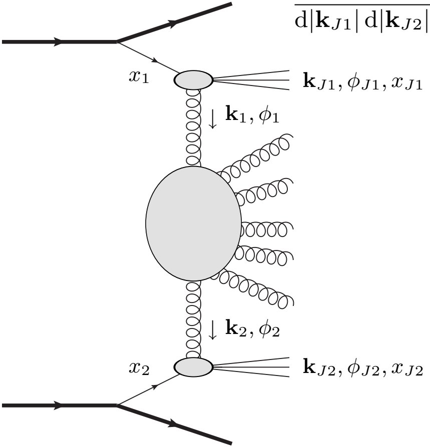
- impact factors are known in some cases at NLL
  - $\gamma^* \to \gamma^*$ at $t = 0$ (Bartels, Colferai, Gieseke, Kyrieleis, Qiao; Balitski, Chirilli)
  - forward jet production (Bartels, Colferai, Vacca; Caporale, Ivanov, Murdaca, Papa, Perri; Chachamis, Hentschinski, Madrigal, Sabio Vera)
  - inclusive production of a pair of hadrons separated by a large interval of rapidity (Ivanov, Papa)
  - $\gamma_L^* \to \rho_L$ in the forward limit (Ivanov, Kotsky, Papa)

## Mueller-Navelet jets

- Consider two jets (hadrons flying within a narrow cone) separated by a large rapidity, i.e. each of them almost fly in the direction of the hadron "close" to it, and with very similar transverse momenta
- in a pure LO collinear treatment, these two jets should be emitted back to back at leading order: $\Delta\phi - \pi = 0$ ($\Delta\phi = \phi_1 - \phi_2 =$ relative azimuthal angle) and $k_{\perp 1}$=$k_{\perp 2}$. There is no phase space for (untagged) emission between them



## $k_T$-factorized differential cross section




$$\frac{\mathrm{d}\sigma}{\mathrm{d}|\mathbf{k}_{J1}|\,\mathrm{d}|\mathbf{k}_{J2}|\,\mathrm{d}y_{J1}\,\mathrm{d}y_{J2}} = \int \mathrm{d}\phi_{J1}\,\mathrm{d}\phi_{J2} \int \mathrm{d}^2\mathbf{k}_1\,\mathrm{d}^2\mathbf{k}_2$$

$$\times\, \Phi(\mathbf{k}_{J1}, x_{J1}, -\mathbf{k}_1)$$

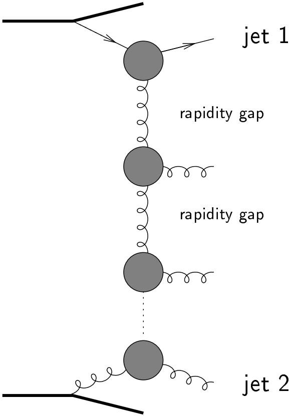
$$\times\, G(\mathbf{k}_1, \mathbf{k}_2, \hat{s})$$

$$\times\, \Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2)$$

with $\Phi(\mathbf{k}_{J2}, x_{J2}, \mathbf{k}_2) = \int \mathrm{d}x_2\, f(x_2) V(\mathbf{k}_2, x_2)$ $\quad f \equiv$ PDF $\quad x_J = \frac{|\mathbf{k}_J|}{\sqrt{s}} e^{y_J}$

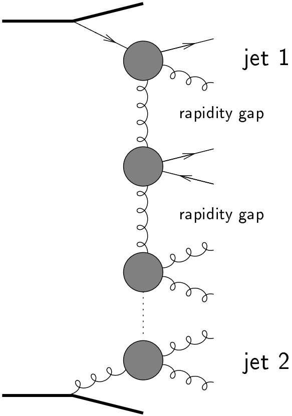
# Mueller-Navelet jets: LL vs NLL

LL BFKL




$$\sum(\alpha_s \ln s)^n$$

NLL BFKL




$$\sum(\alpha_s \ln s)^n + \alpha_s \sum(\alpha_s \ln s)^n$$

In the following we show results for

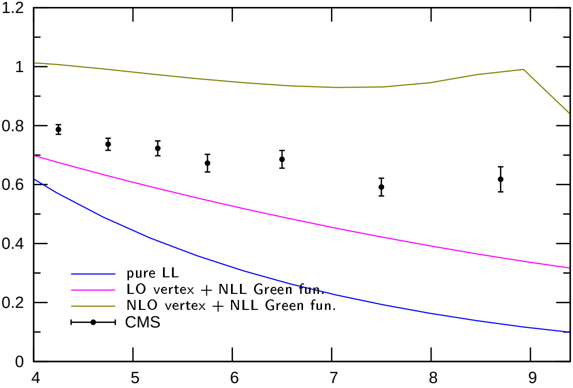
- $\sqrt{s} = 7$ TeV
- $35\,\mathrm{GeV} < |\mathbf{k}_{J1}|\,,|\mathbf{k}_{J2}| < 60\,\mathrm{GeV}$
- $0 < |y_1|\,,|y_2| < 4.7$

These cuts allow us to compare our predictions with the first experimental data on azimuthal correlations of Mueller-Navelet jets at the LHC presented by the CMS collaboration (CMS-PAS-FSQ-12-002)

note: unlike experiments we have to set an upper cut on $|\mathbf{k}_{J1}|$ and $|\mathbf{k}_{J2}|$. We have checked that our results don't depend on this cut significantly.

## Azimuthal correlation $\langle \cos \varphi \rangle$

$$\frac{\mathcal{C}_1}{\mathcal{C}_0} = \langle \cos \varphi \rangle \equiv \langle \cos(\phi_{J1} - \phi_{J2} - \pi) \rangle$$




$Y \equiv |y_1 - y_2|$

$35\,\text{GeV} < |\mathbf{k}_{J1}| < 60\,\text{GeV}$

$35\,\text{GeV} < |\mathbf{k}_{J2}| < 60\,\text{GeV}$

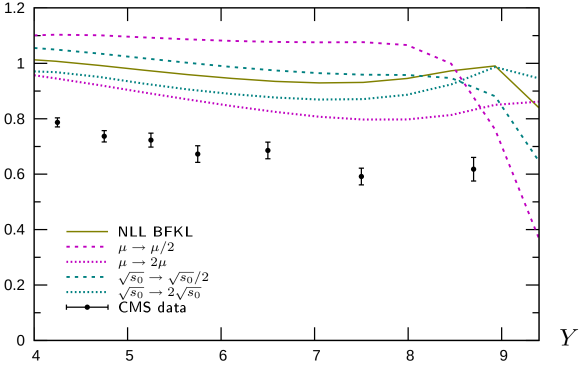
$0 < |y_1| < 4.7$

$0 < |y_2| < 4.7$

The NLO corrections to the jet vertex lead to a large increase of the correlation

## Azimuthal correlation $\langle \cos \varphi \rangle$

$\langle \cos \varphi \rangle \equiv \langle \cos(\phi_{J1} - \phi_{J2} - \pi) \rangle$

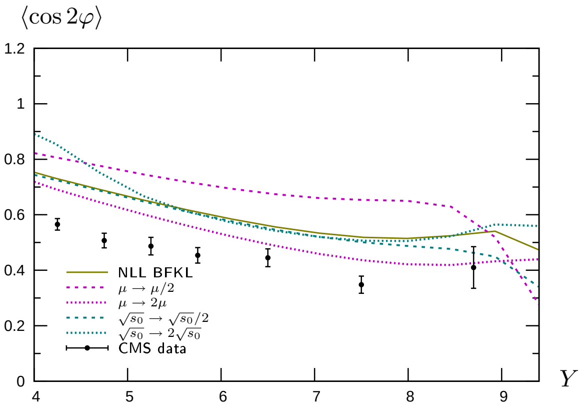



$35\,\mathrm{GeV} < |\mathbf{k}_{J1}| < 60\,\mathrm{GeV}$
$35\,\mathrm{GeV} < |\mathbf{k}_{J2}| < 60\,\mathrm{GeV}$

$0 < |y_1| < 4.7$
$0 < |y_2| < 4.7$

- NLL BFKL predicts a too small decorrelation
- The NLL BFKL calculation is still rather dependent on the scales, especially the renormalization / factorization scale

Azimuthal correlation $\langle \cos 2\varphi \rangle$

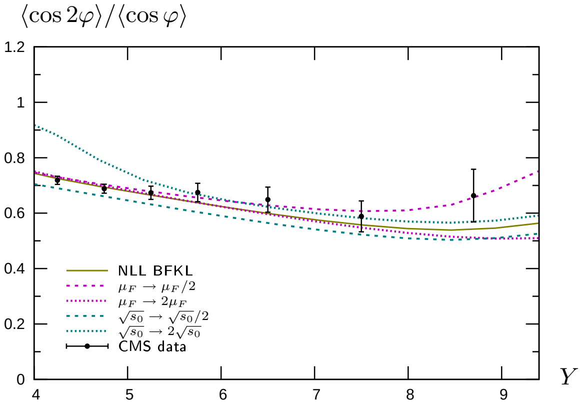



$35\,\mathrm{GeV} < |\mathbf{k}_{J1}| < 60\,\mathrm{GeV}$
$35\,\mathrm{GeV} < |\mathbf{k}_{J2}| < 60\,\mathrm{GeV}$

$0 < |y_1| < 4.7$
$0 < |y_2| < 4.7$

- The agreement with data is a little better for $\langle \cos 2\varphi \rangle$ but still not very good
- This observable is also very sensitive to the scales

## Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$

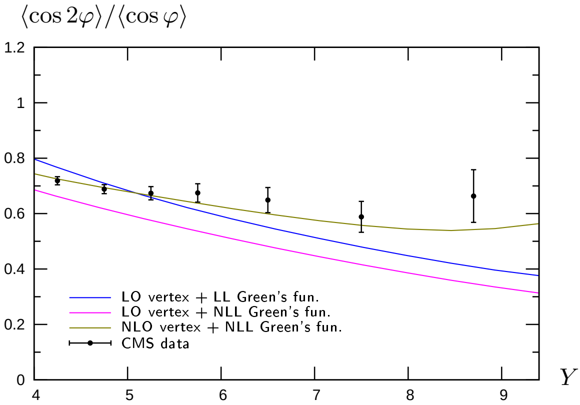



$35\ \text{GeV} < |\mathbf{k}_{J1}| < 60\ \text{GeV}$
$35\ \text{GeV} < |\mathbf{k}_{J2}| < 60\ \text{GeV}$

$0 < |y_1| < 4.7$
$0 < |y_2| < 4.7$

- This observable is more stable with respect to the scales than the previous ones
- The agreement with data is good across the full $Y$ range

Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$

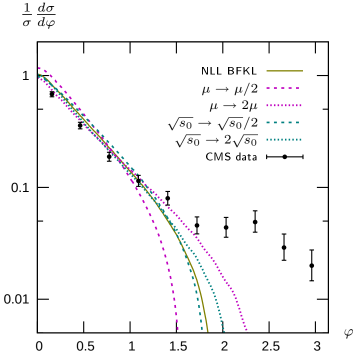



$35\,\mathrm{GeV} < |\mathbf{k}_{J1}| < 60\,\mathrm{GeV}$
$35\,\mathrm{GeV} < |\mathbf{k}_{J2}| < 60\,\mathrm{GeV}$

$0 < |y_1| < 4.7$
$0 < |y_2| < 4.7$

It is necessary to include the NLO corrections to the jet vertex to reproduce the behavior of the data at large $Y$

## Azimuthal distribution (integrated over $6 < Y < 9.4$)




- Our calculation predicts a too large value of $\frac{1}{\sigma}\frac{d\sigma}{d\varphi}$ for $\varphi \lesssim \frac{\pi}{2}$ and a too small value for $\varphi \gtrsim \frac{\pi}{2}$
- It is not possible to describe the data even when varying the scales by a factor of 2

- The agreement of our calculation with the data for $\langle\cos 2\varphi\rangle/\langle\cos\varphi\rangle$ is good and quite stable with respect to the scales
- The agreement for $\langle\cos n\varphi\rangle$ and $\frac{1}{\sigma}\frac{d\sigma}{d\varphi}$ is not very good and very sensitive to the choice of the renormalization scale $\mu_R$
- An all-order calculation would be independent of the choice of $\mu_R$. This feature is lost if we truncate the perturbative series
  $\Rightarrow$ How to choose the renormalization scale?
  'Natural scale': sometimes the typical momenta in a loop diagram are different from the natural scale of the process

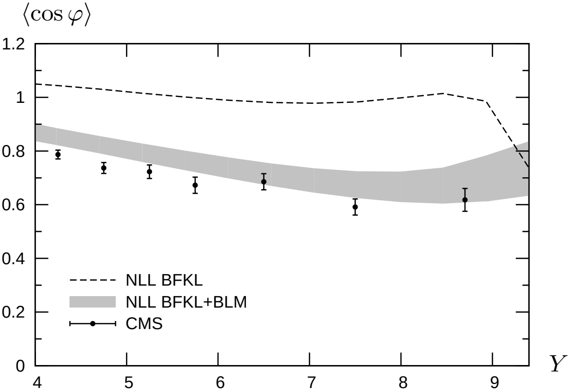
We decided to use the Brodsky-Lepage-Mackenzie (BLM) procedure to fix the renormalization scale

The Brodsky-Lepage-Mackenzie (BLM) procedure resums the self-energy corrections to the gluon propagator at one loop into the running coupling.

First attempts to apply BLM scale fixing to BFKL processes lead to problematic results. Brodsky, Fadin, Kim, Lipatov and Pivovarov suggested that one should first go to a physical renormalization scheme like MOM and then apply the 'traditional' BLM procedure, i.e. identify the $\beta_0$ dependent part and choose $\mu_R$ such that it vanishes.

We followed this prescription for the full amplitude at NLL.

Azimuthal correlation $\langle \cos\varphi \rangle$

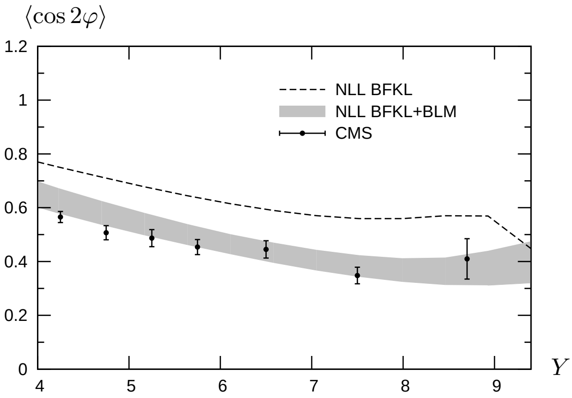



$35\,\mathrm{GeV} < |\mathbf{k}_{J1}| < 60\,\mathrm{GeV}$
$35\,\mathrm{GeV} < |\mathbf{k}_{J2}| < 60\,\mathrm{GeV}$

$0 < |y_1| < 4.7$
$0 < |y_2| < 4.7$

Using the BLM scale setting, the agreement with data becomes much better

## Azimuthal correlation $\langle \cos 2\varphi \rangle$




$35\,\mathrm{GeV} < |\mathbf{k}_{J1}| < 60\,\mathrm{GeV}$

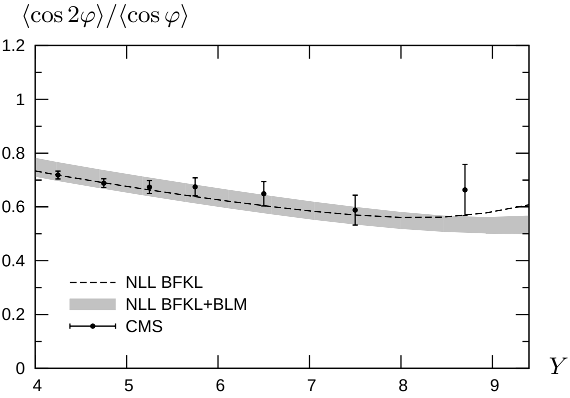
$35\,\mathrm{GeV} < |\mathbf{k}_{J2}| < 60\,\mathrm{GeV}$

$0 < |y_1| < 4.7$

$0 < |y_2| < 4.7$

Using the BLM scale setting, the agreement with data becomes much better

## Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$




$35\ \text{GeV} < |\mathbf{k}_{J1}| < 60\ \text{GeV}$

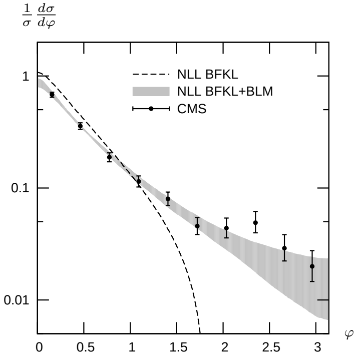
$35\ \text{GeV} < |\mathbf{k}_{J2}| < 60\ \text{GeV}$

$0 < |y_1| < 4.7$

$0 < |y_2| < 4.7$

Because it is much less dependent on the scales, the observable $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ is almost not affected by the BLM procedure and is still in good agreement with the data

Azimuthal distribution (integrated over $6 < Y < 9.4$)




With the BLM scale setting the azimuthal distribution is in good agreement with the data across the full $\varphi$ range.

Using the BLM scale setting:

- The agreement $\langle \cos n\varphi \rangle$ with the data becomes much better
- The agreement for $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ is still good and unchanged as this observable is weakly dependent on $\mu_R$
- The azimuthal distribution is in much better agreement with the data

But the configuration chosen by CMS with $\mathbf{k}_{J\min 1} = \mathbf{k}_{J\min 2}$ does not allow us to compare with a fixed-order $\mathcal{O}(\alpha_s^3)$ treatment (i.e. without resummation)

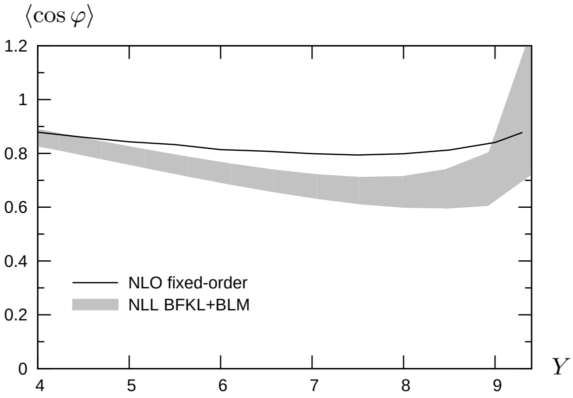
These calculations are unstable when $\mathbf{k}_{J\min 1} = \mathbf{k}_{J\min 2}$ because the cancellation of some divergencies is difficult to obtain numerically

In this section we choose the cuts as

- $35\,\mathrm{GeV} < |\mathbf{k}_{J1}|\,, |\mathbf{k}_{J2}| < 60\,\mathrm{GeV}$
- $50\,\mathrm{GeV} < \mathrm{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$
- $0 < |y_1|\,, |y_2| < 4.7$

And we compare our results with the NLO fixed-order code Dijet (Aurenche, Basu, Fontannaz) in the same configuration

## Azimuthal correlation $\langle \cos \varphi \rangle$




$35\,\text{GeV} < |\mathbf{k}_{J1}| < 60\,\text{GeV}$

$35\,\text{GeV} < |\mathbf{k}_{J2}| < 60\,\text{GeV}$

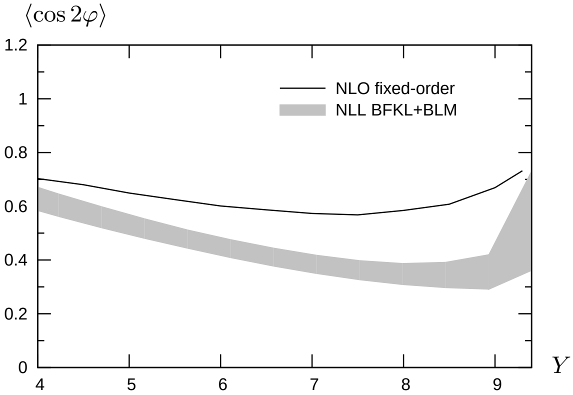
$50\,\text{GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$

$0 < |y_1| < 4.7$

$0 < |y_2| < 4.7$

The NLO fixed-order and NLL BFKL+BLM calculations are very close

## Azimuthal correlation $\langle \cos 2\varphi \rangle$

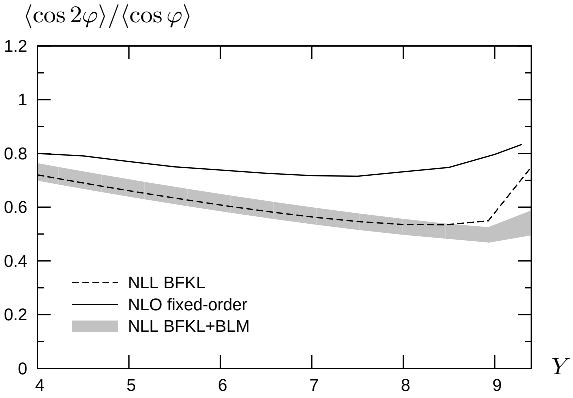



$35\,\text{GeV} < |\mathbf{k}_{J1}| < 60\,\text{GeV}$
$35\,\text{GeV} < |\mathbf{k}_{J2}| < 60\,\text{GeV}$
$50\,\text{GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$

$0 < |y_1| < 4.7$
$0 < |y_2| < 4.7$

The BLM procedure leads to a sizable difference between NLO fixed-order and NLL BFKL+BLM

Azimuthal correlation $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$

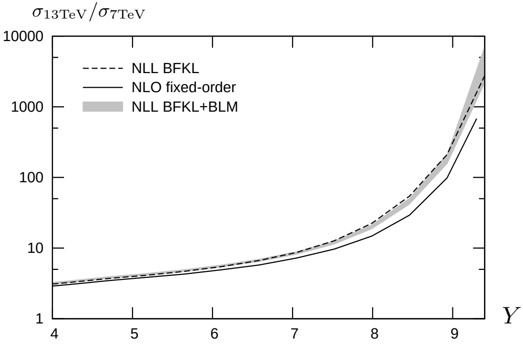



$35\,\text{GeV} < |\mathbf{k}_{J1}| < 60\,\text{GeV}$
$35\,\text{GeV} < |\mathbf{k}_{J2}| < 60\,\text{GeV}$
$50\,\text{GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$

$0 < |y_1| < 4.7$
$0 < |y_2| < 4.7$

Using BLM or not, there is a sizable difference between BFKL and fixed-order

## Cross section: 13 TeV vs. 7 TeV




$35\,\text{GeV} < |\mathbf{k}_{J1}| < 60\,\text{GeV}$
$35\,\text{GeV} < |\mathbf{k}_{J2}| < 60\,\text{GeV}$
$50\,\text{GeV} < \text{Max}(|\mathbf{k}_{J1}|, |\mathbf{k}_{J2}|)$

$0 < |y_1| < 4.7$
$0 < |y_2| < 4.7$

- In a BFKL treatment, a strong rise of the cross section with increasing energy is expected.
- This rise is faster than in a fixed-order treatment

It is necessary to have $\mathbf{k}_{J\text{min}1} \neq \mathbf{k}_{J\text{min}2}$ for comparison with fixed order calculations but this can be problematic for BFKL because of energy-momentum conservation

There is no strict energy-momentum conservation in BFKL

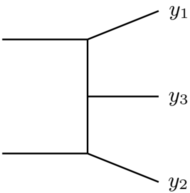
This was studied at LO by Del Duca and Schmidt. They introduced an effective rapidity $Y_{\text{eff}}$ defined as

$$Y_{\text{eff}} \equiv Y \frac{\sigma^{2\to 3}}{\sigma^{\text{BFKL},\mathcal{O}(\alpha_{\text{s}}^3)}}$$

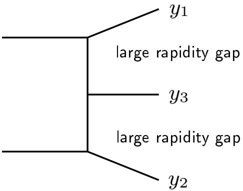
When one replaces $Y$ by $Y_{\text{eff}}$ in the expression of $\sigma^{\text{BFKL}}$ and truncates to $\mathcal{O}(\alpha_s^3)$, the exact $2 \to 3$ result is obtained

We follow the idea of Del Duca and Schmidt, adding the NLO jet vertex contribution:

exact $2 \to 3$



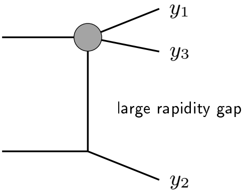

BFKL



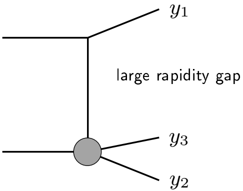

1 emission from the Green's function + LO jet vertex

we have to take into account these additional $\mathcal{O}(\alpha_s^3)$ contributions:

+

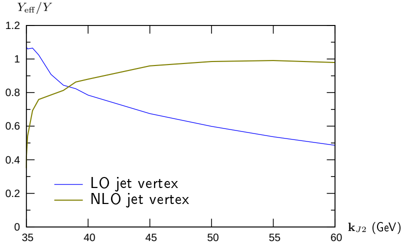



+




no emission from the Green's function + NLO jet vertex

Variation of $Y_{\text{eff}}/Y$ as a function of $\mathbf{k}_{J2}$ for fixed $\mathbf{k}_{J1} = 35$ GeV (with $\sqrt{s} = 7$ TeV, $Y = 8$):




- With the LO jet vertex, $Y_{\text{eff}}$ is much smaller than $Y$ when $\mathbf{k}_{J1}$ and $\mathbf{k}_{J2}$ are significantly different
- This is the region important for comparison with fixed order calculations
- The improvement coming from the NLO jet vertex is very large in this region
- For $\mathbf{k}_{J1} = 35$ GeV and $\mathbf{k}_{J2} = 50$ GeV, typical of the values we used for comparison with fixed order, we get $\frac{Y_{\text{eff}}}{Y} \simeq 0.98$ at NLO vs. $\sim 0.6$ at LO

# Conclusions

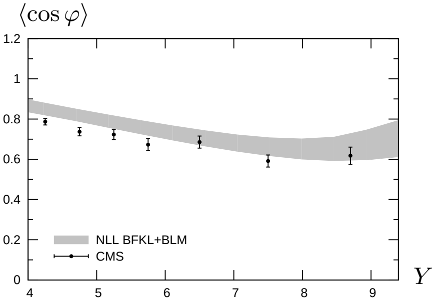
- We studied Mueller-Navelet jets at full (vertex + Green's function) NLL accuracy and compared our results with the first data from the LHC
- The agreement with CMS data at $7$ TeV is greatly improved by using the BLM scale fixing procedure
- $\langle \cos 2\varphi \rangle / \langle \cos \varphi \rangle$ is almost not affected by BLM and shows a clear difference between NLO fixed-order and NLL BFKL in an asymmetric configuration

  Energy-momentum conservation seems to be less severely violated with the NLO jet vertex
- We did the same analysis at $13$ TeV:
  - Azimuthal decorrelations don't show a very different behavior at 13 TeV compared to 7 TeV
  - NLL BFKL predicts a stronger rise of the cross section with increasing energy than a NLO fixed-order calculation

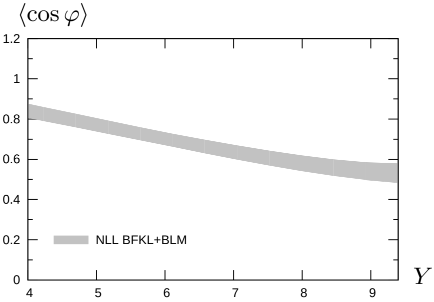
  A measurement of the cross section at $\sqrt{s} = 7$ or $8$ TeV would be needed to test this

Dziękuję za uwagę !

Azimuthal correlation $\langle \cos \varphi \rangle$



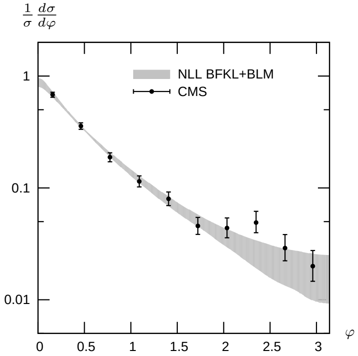

$\sqrt{s} = 7$ TeV



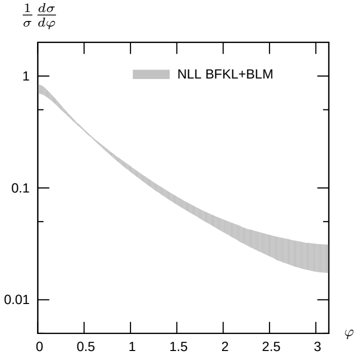

$\sqrt{s} = 13$ TeV

The behavior is similar at 13 TeV and at 7 TeV

Azimuthal distribution (integrated over $6 < Y < 9.4$)




$\sqrt{s} = 7$ TeV

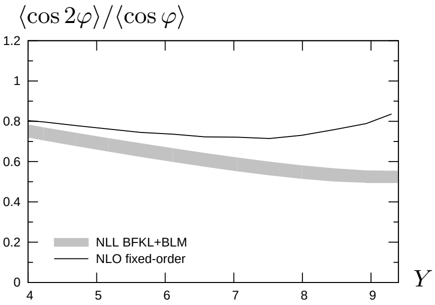



$\sqrt{s} = 13$ TeV

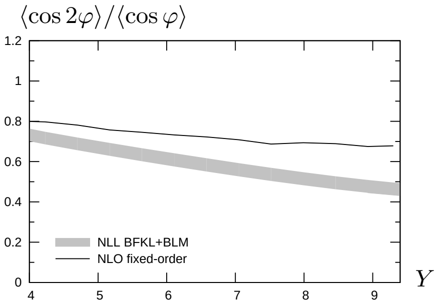
The behavior is similar at 13 TeV and at 7 TeV

Azimuthal correlation $\langle\cos 2\varphi\rangle/\langle\cos\varphi\rangle$

(asymmetric configuration)



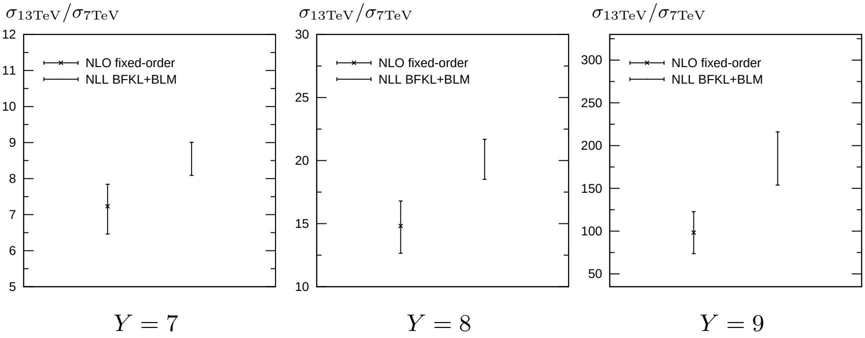

$\sqrt{s} = 7$ TeV




$\sqrt{s} = 13$ TeV

The difference between BFKL and fixed-order is smaller at $13$ TeV than at $7$ TeV

## Cross section



It is useful to define the coefficients $\mathcal{C}_n$ as

$$\mathcal{C}_n \equiv \int \mathrm{d}\phi_{J1}\,\mathrm{d}\phi_{J2}\,\cos\left(n(\phi_{J1}-\phi_{J2}-\pi)\right) \times \int \mathrm{d}^2\mathbf{k}_1\,\mathrm{d}^2\mathbf{k}_2\,\Phi(\mathbf{k}_{J1},x_{J1},-\mathbf{k}_1)\,G(\mathbf{k}_1,\mathbf{k}_2,\hat{s})\,\Phi(\mathbf{k}_{J2},x_{J2},\mathbf{k}_2)$$

- $n=0 \implies$ differential cross-section

$$\mathcal{C}_0 = \frac{\mathrm{d}\sigma}{\mathrm{d}|\mathbf{k}_{J1}|\,\mathrm{d}|\mathbf{k}_{J2}|\,\mathrm{d}y_{J1}\,\mathrm{d}y_{J2}}$$

- $n>0 \implies$ azimuthal decorrelation

$$\frac{\mathcal{C}_n}{\mathcal{C}_0} = \langle\cos\left(n(\phi_{J,1}-\phi_{J,2}-\pi)\right)\rangle \equiv \langle\cos(n\varphi)\rangle$$

- sum over $n \implies$ azimuthal distribution

$$\frac{1}{\sigma}\frac{d\sigma}{d\varphi} = \frac{1}{2\pi}\left\{1+2\sum_{n=1}^{\infty}\cos(n\varphi)\,\langle\cos(n\varphi)\rangle\right\}$$